

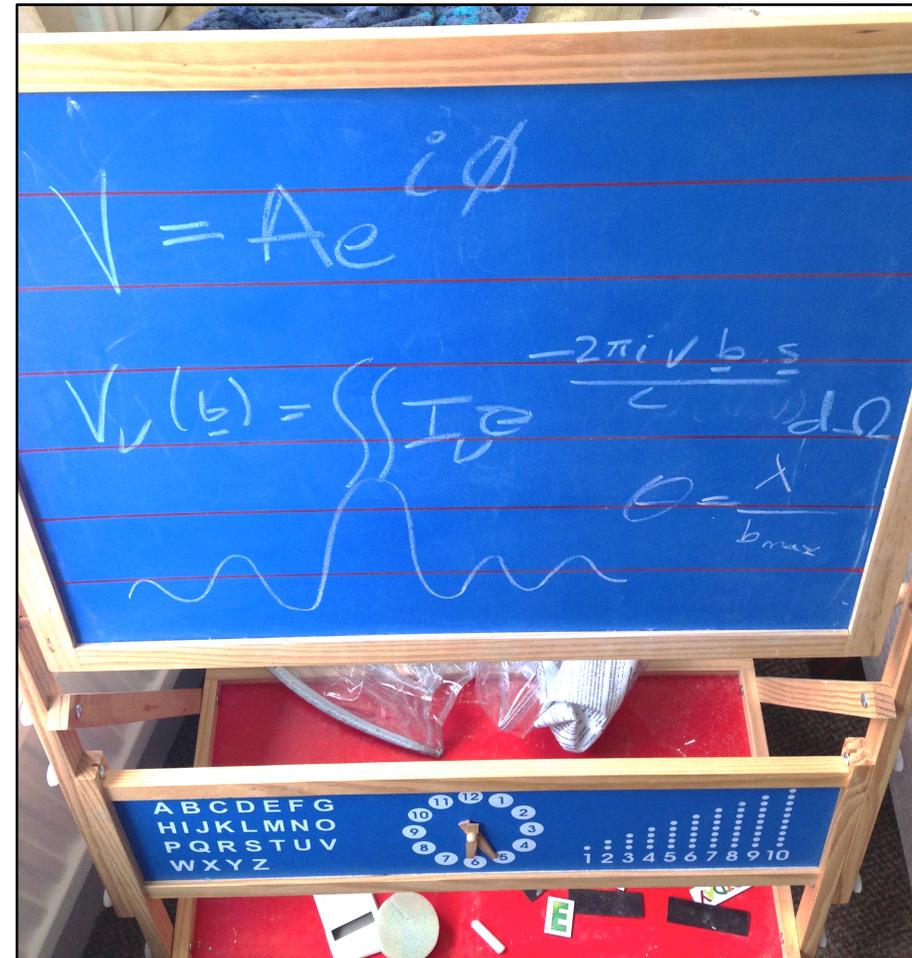
Interferometer fundamentals

Adam Avison



Outline

- Why use an interferometer?
- Two element interferometer
- uv -coverage
- A bit on calibration
- Imaging data and Cleaning



Why use an interferometer?

- What do astronomers want?

- High resolution

$$\theta \approx \frac{\lambda}{D}$$

- High sensitivity

$$S_{\nu, rms} = \frac{2kT_{sys}}{A_e \sqrt{\tau \Delta\nu}}$$



Resolution

Imagine we want Hubble resolution:

$$D=2.4\text{m}, \lambda=600\text{nm} \rightarrow \theta=0.05\text{arcsec}$$

At radio wavelengths, say 4.5cm (6.7GHz):

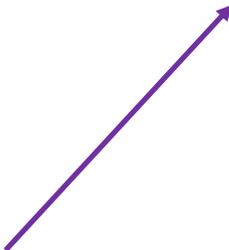
D would need to be $\sim 185\text{km}$

Sensitivity

Basically, the bigger the better!

$$S_{\nu, rms} = \frac{2kT_{sys}}{A_e \sqrt{\tau \Delta\nu}}$$

Effective area of your dish



How do we get it?

- Interferometry

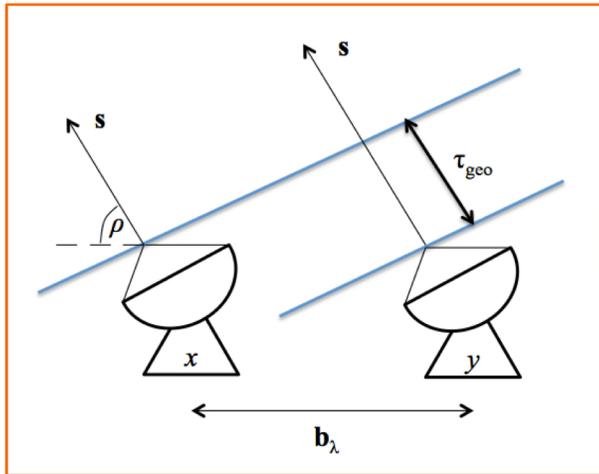
$$\theta \approx \frac{\lambda}{b_{\max}}$$

$$\Delta S_{rms} = \frac{2kT_{sys}}{A_e \sqrt{N(N-1)\tau \Delta\nu}}$$



ALMA, 54 12m dishes and 12 7m dishes spread over 14km

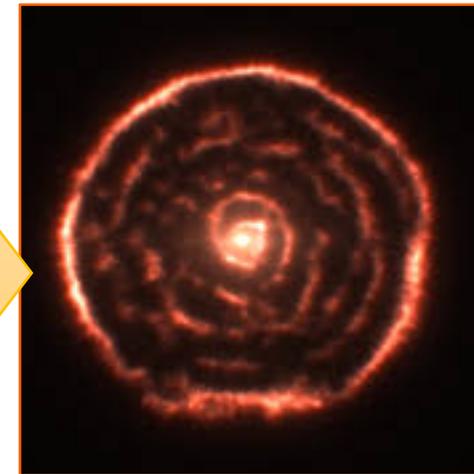
Most peoples first impression of interferometry



Dishes



Dark Arts



Pretty pictures

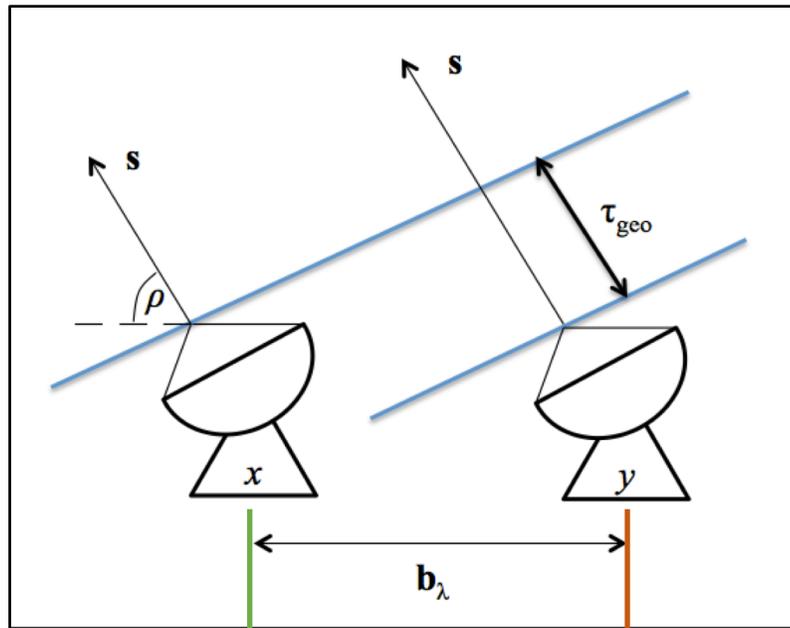
... hopefully by the end of this hour we'll have cleared up the bit in the middle

The two element interferometer

The two element interferometer

What are we trying to do?

- Recreate the sky brightness distribution of astrophysical sources.



Delay between wavefronts arriving at x then y :

$$\tau_{geo} = \frac{\mathbf{b} \cdot \mathbf{s}}{c} = \frac{bs \cos \rho}{c}$$

$$x(t) = v_1 \cos 2\pi\nu t$$

$$y(t) = v_2 \cos 2\pi\nu(t + \tau_{geo})$$
 Receiver outputs

$$R_{x,y}(\tau_{geo}) = x \otimes y = X(\nu)Y^*(\nu)$$
 Correlator's function

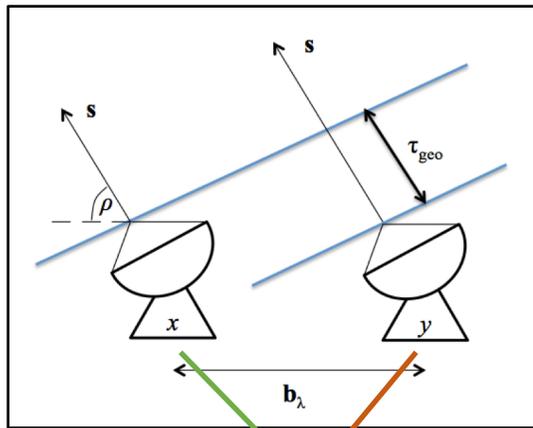
Do the maths

$$R_{x,y}(\tau_{geo}) = v_1 v_2 \cos 2\pi\nu\tau_{geo}$$
 Correlator output

That's all well and good but how does this


$$R_{x,y}(\tau_{geo}) = v_1 v_2 \cos 2\pi \nu \tau_{geo}$$

tell us anything about the sky
brightness distribution?



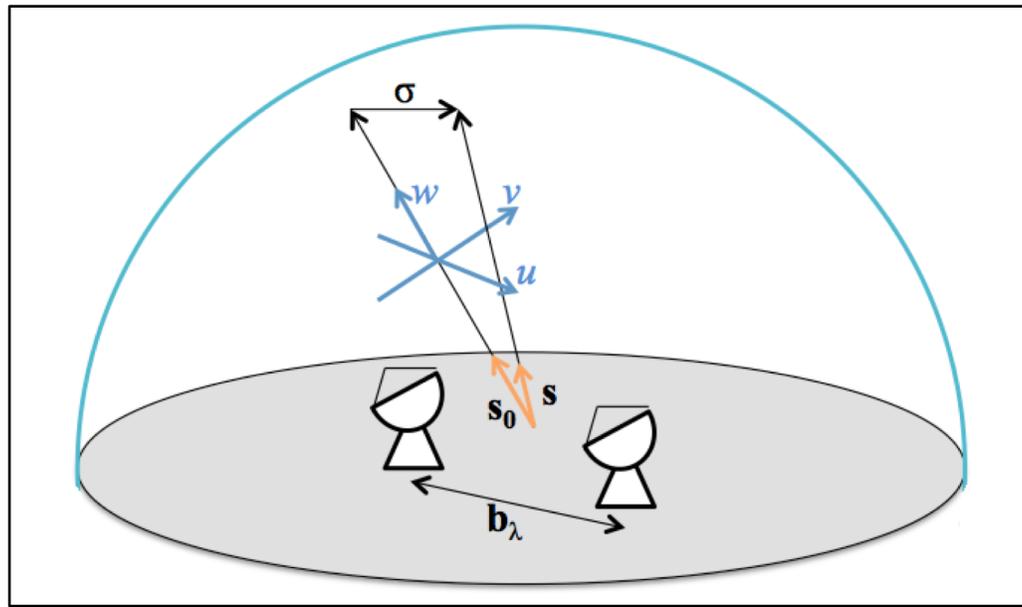
$$R_{x,y}(\tau_{geo}) = v_1 v_2 \cos 2\pi \nu \tau_{geo}$$

v_1 and v_2 , the voltage outputs of x & y are directly related to:

- The brightness distribution, $I(\mathbf{s})$, of the astronomical object
- as seen over solid angle $d\Omega$
- and $A(\mathbf{s})$ the area of the dish we use to observe it.

Leading to ...

$$R_{x,y}(\tau_{geo}) = \Delta \nu \int A(\mathbf{s}) I(\mathbf{s}) \cos 2\pi \mathbf{b}_\lambda \cdot \mathbf{s} d\Omega$$



Adding in a bit more reality...

- The vector \mathbf{s} is comprised of the addition of \mathbf{s}_0 and σ (so $\mathbf{s} = \mathbf{s}_0 + \sigma$).
- We set τ_{geo} to zero with instrumental delays
- Meaning all delays in the data are from the vector σ

We then define the Complex Visibility as:

$$V \equiv |V| e^{i\phi_V} = \int A(\sigma) I(\sigma) e^{-i2\pi \mathbf{b}_\lambda \cdot \sigma} d\Omega$$

which is rather nice as V is the Fourier transform of I .

Relating the visibility equation to the correlator output gives

$$R_{x,y}(\tau_{geo}) = \Delta\nu \int A(\mathbf{s})I(\mathbf{s})\cos 2\pi\mathbf{b}_\lambda \cdot \mathbf{s} \, d\Omega$$

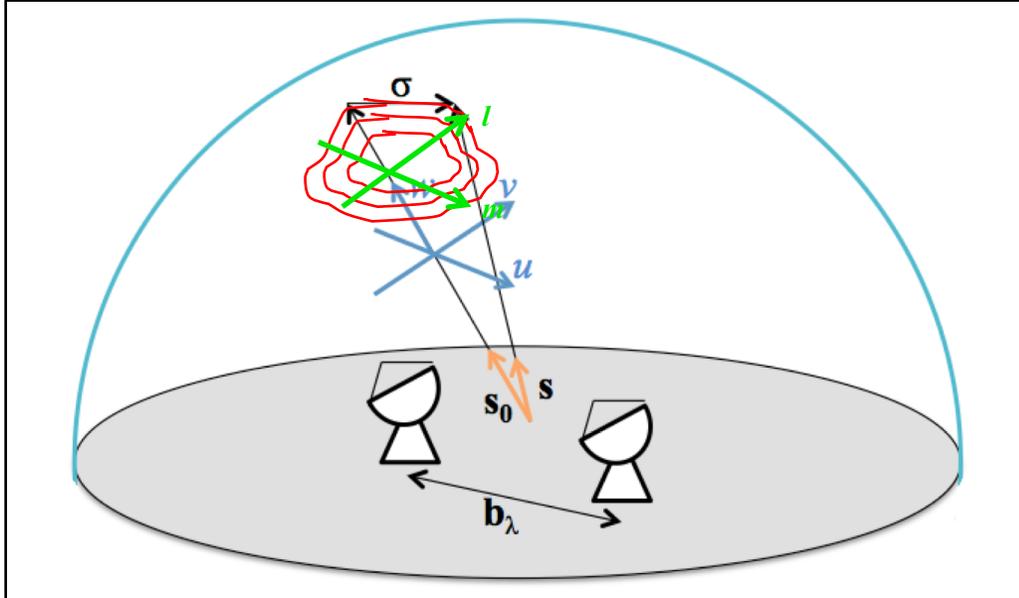
$$V \equiv |V|e^{i\phi_V} = \int A(\sigma)I(\sigma)e^{-i2\pi\mathbf{b}_\lambda \cdot \sigma} \, d\Omega$$

$$R_{x,y} = A_0 |V| \Delta\nu \cos(2\pi\nu \mathbf{b}_\lambda \cdot \mathbf{s}_0 - \phi_V)$$

Known!*

* After proper calibration

A coordinate system for interferometry



We define u and v , as E-W and N-S positions w.r.t w axis which is parallel to s_0 .

l and m as direction cosines of \mathbf{s} we can write the visibility equation as:

$$V(u, v) = \int A(l, m) I(l, m) e^{-i2\pi(ul+vm)} \frac{dl dm}{\sqrt{1-l^2-m^2}}$$

Given l and m are small the small angle approx applies and $V(u, v)$ becomes a direct Fourier transform of $I(x, y)$

In its full **gory** glory the... Measurement Equation

$$V_{ij} = M_{ij} B_{ij} G_{ij} D_{ij} \int E_{ij} P_{ij} T_{ij} F_{ij} S I_v(l, m) e^{-i2\pi(u_{ij}l + v_{ij}m)} \frac{dl dm}{\sqrt{1-l^2-m^2}} + Q_{ij}$$

V_{ij} = What we measure

I_v = What we want

Q_{ij} = additive errors

S = maps I to polarisation

i, j = telescope pair

M_{ij} = Multiplicative baselines errors

B_{ij} = Bandpass reponse

G_{ij} = Gerenalised electronic gain

D_{ij} = polarisation leakage

E_{ij} = Antenna voltage pattern

P_{ij} = paralatic angle

T_{ij} = Tropospheric effects

Green= vectors

Blue= Scalars

Red= Part of the Jones Matrix

What we measure

Correlator output

$$R_{x,y} = A_0 |V| \Delta \nu \cos(2\pi \nu \mathbf{b}_\lambda \cdot \mathbf{s}_0 - \phi_V)$$

Eq. 1

Visibility equation

$$V(u, v) = \int A(l, m) I(l, m) e^{-i2\pi(ul+vm)} \frac{dldm}{\sqrt{1-l^2-m^2}}$$

Eq. 2

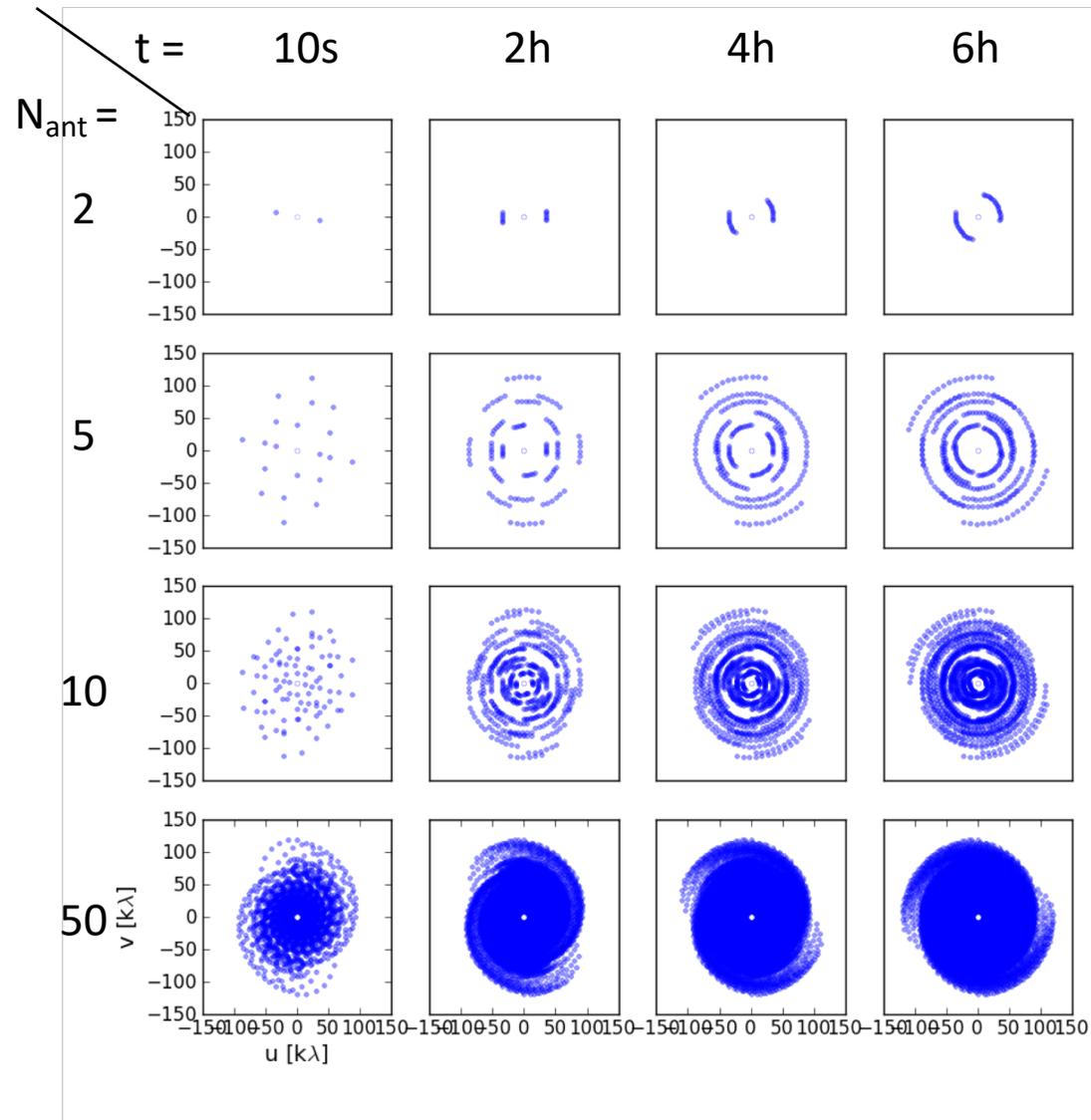
Dirty image

$$I^D(l, m) = \iint_{-\infty}^{\infty} S(u, v) V(u, v) e^{2\pi i(ul+vm)} dudv$$

Eq. 3

uv-coverage

Filling the uv -plane



uv -coverage of an interferometer set out in a logarithmic spiral pattern comprised of two, five, ten and fifty antennas (top to bottom) and observing for 10 s, 2, 4, and 6 h (left to right).

ASIDE: Antenna spacing to u, v, w

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{1}{\lambda} \begin{pmatrix} \sin H_0 & \cos H_0 & 0 \\ -\sin \delta_0 \cos H_0 & \sin \delta_0 \sin H_0 & \cos \delta_0 \\ \cos \delta_0 \cos H_0 & -\cos \delta_0 \sin H_0 & \sin \delta_0 \end{pmatrix} \begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix}$$

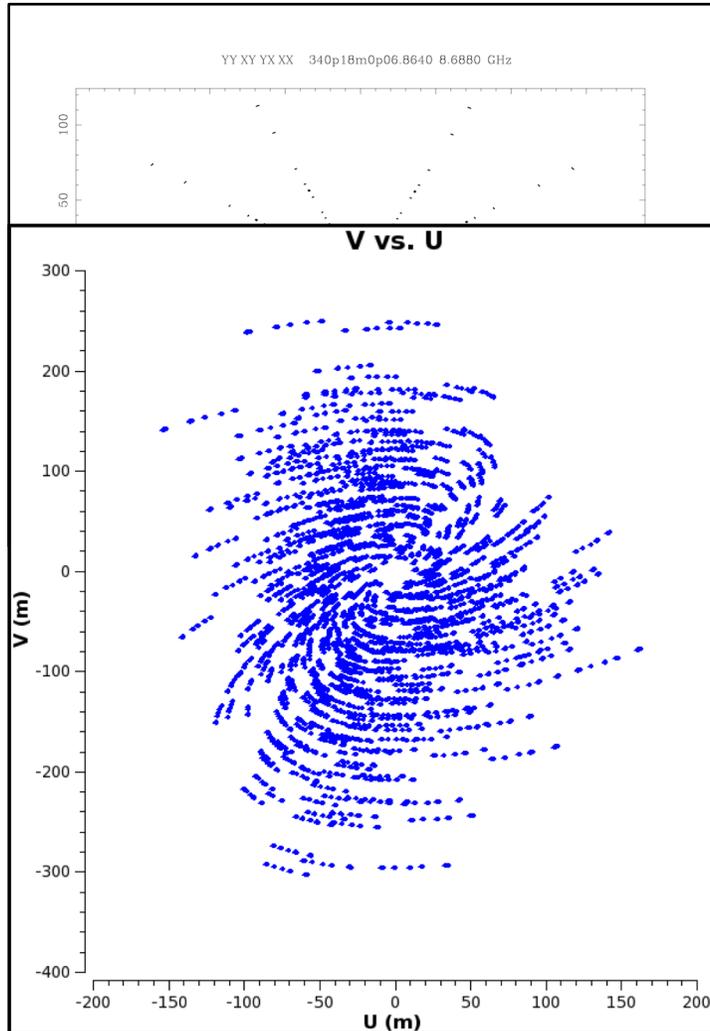
L_x, L_y, L_z = antenna coordinate differences

H_0, δ_0 = hour-angle and declination of the phase reference position

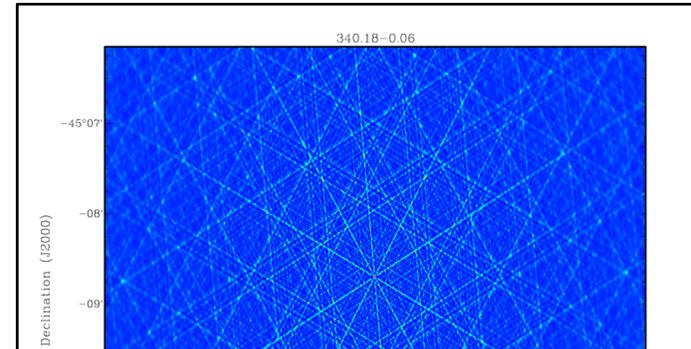
λ = central frequency of observation

Filling the uv -plane

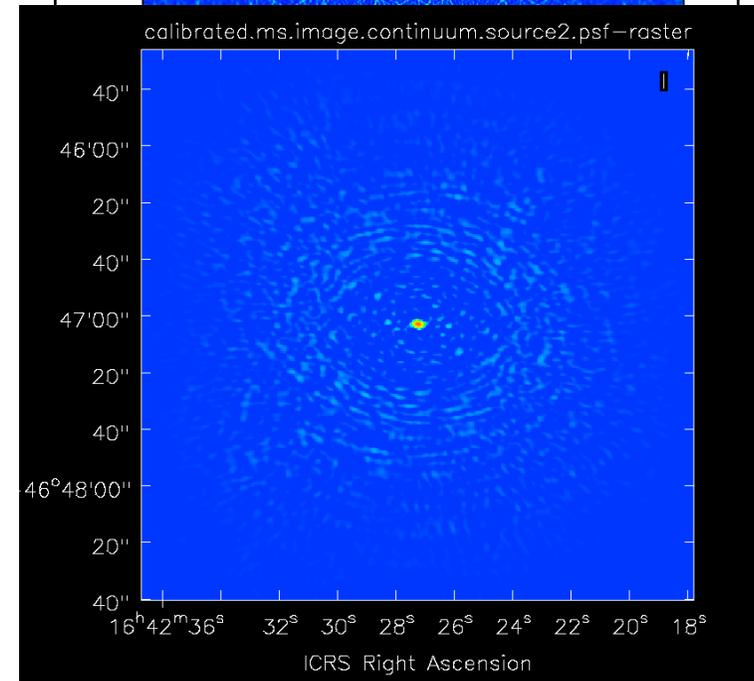
We want to fill the uv -plane because the uv -coverage is the FT of the synthesised beam, B . The greater the uv -coverage the better behaved the sidelobes are.



6 dishes
(~15 mins)



42 dishes
(~15 mins)



Filling the uv -plane

$$V(u, v) = \int A(l, m) I(l, m) e^{-i2\pi(ul+vm)} \frac{dldm}{\sqrt{1-l^2-m^2}}$$

For each antenna pair at each integration interval we get one uv measurement.

To optimize uv -coverage, thus giving us a 'nicer' synthesized beam we would ideally have:

- 1) A large number of dishes, thus more antenna pairs ($N(N-1)$)
- 2) Greater time on source, more uv -points from the antennas we have.
- 3) An array configuration with a larger number of unique antenna spacings.

Visual example

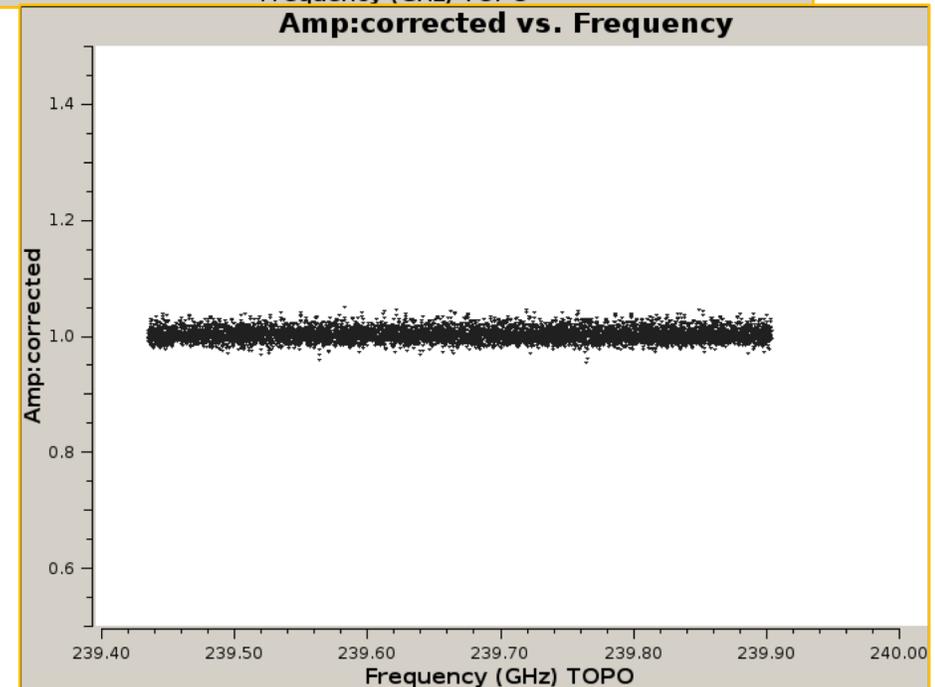
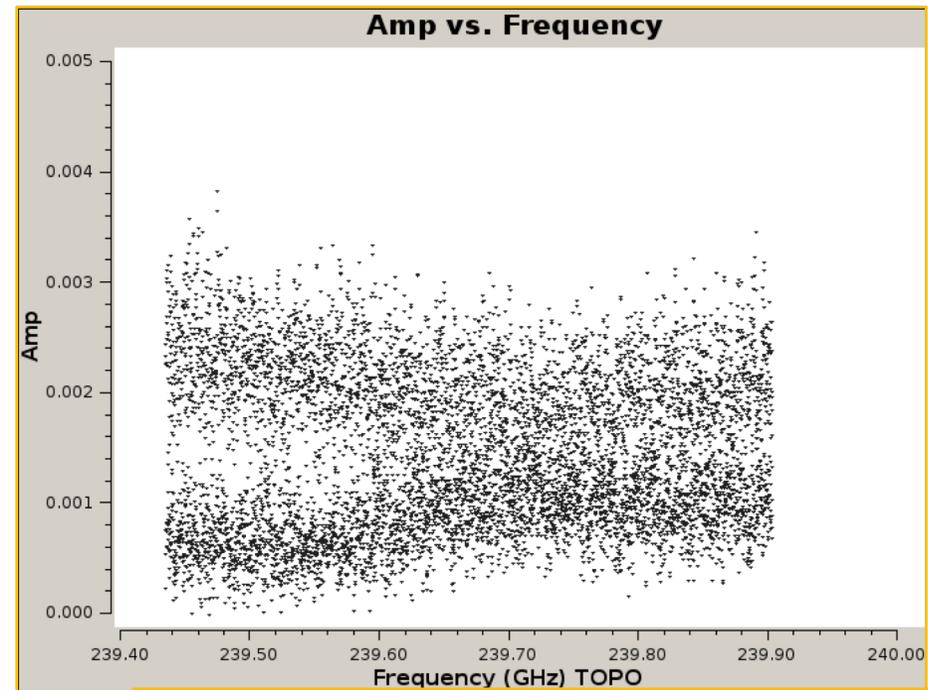
A bit about calibration...

All interferometers have the following steps in common.

- 1) Apriori steps. Some selection of system specific steps which need taking into account before standard calibration. *We'll ignore these*
- 2) Bandpass calibration
- 3) Flux calibration
- 4) Gain calibration / Amp & phase calibration

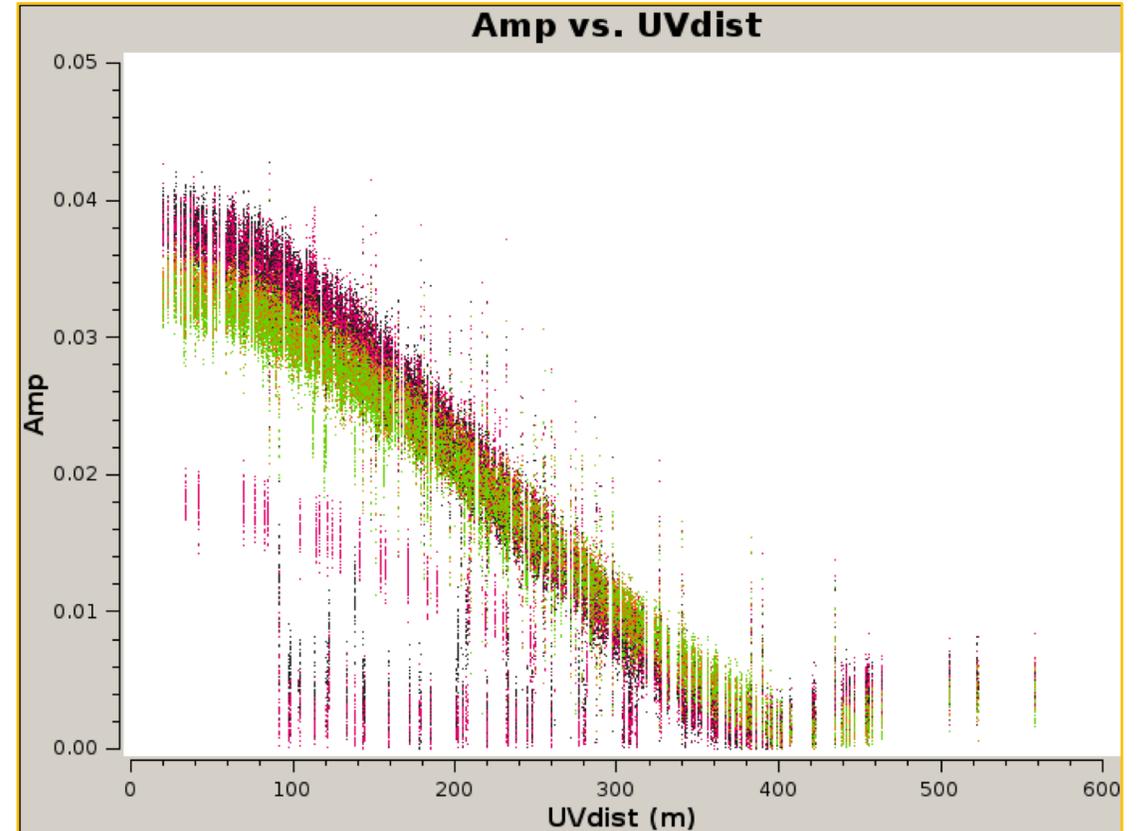
Bandpass calibration

- Antenna based calibration. So solutions are per antenna w.r.t reference antenna.
- Each receiver will have a unique response to emission at a given ν .
- But we know our BP cal has a flat spectrum (or known spectral index).
- Calibrator needn't be close to target on sky.



Flux scaling

- The visibility amplitudes which come out of the correlator have an arbitrary scaling.
- To convert these into physically meaningful values we observe a source of known flux to which we can “bootstrap” the amplitudes values to.
- Calibrator needn't be close to the source. (But should avoid being low in the sky).

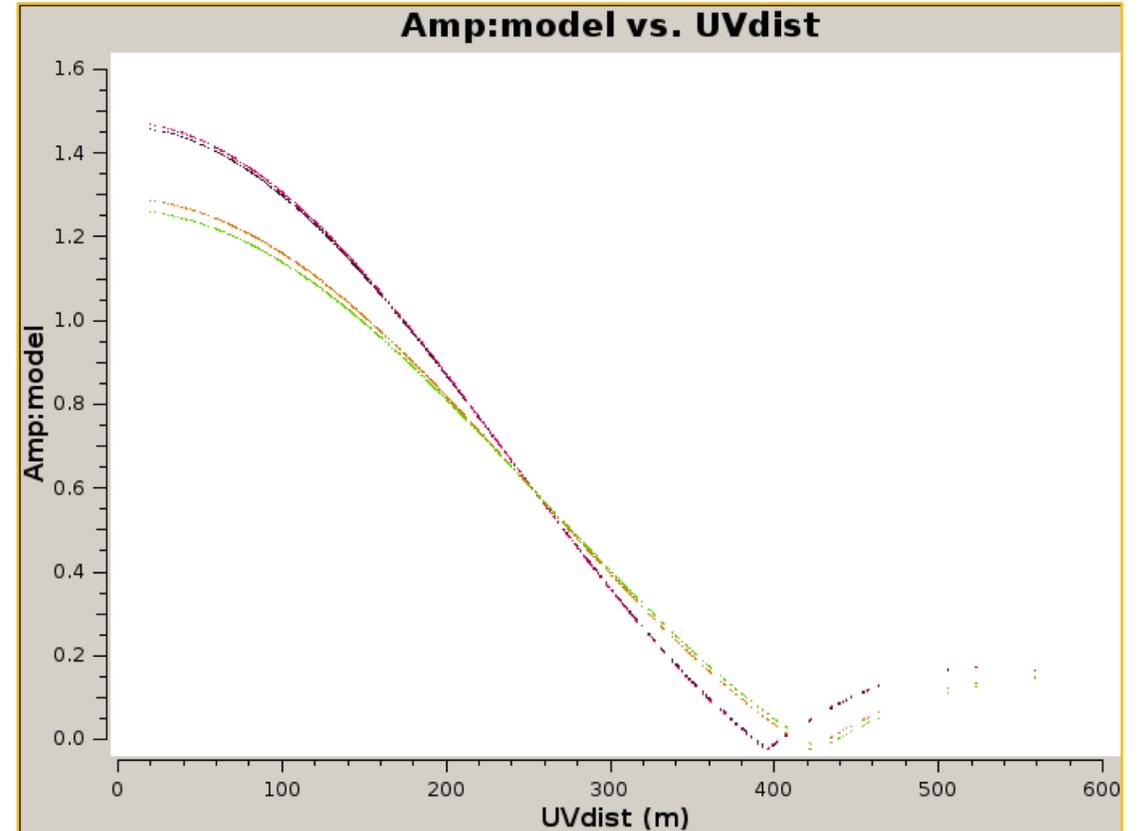


ALMA data of Titan for a given dataset (*some bad data here!*).

Y-axis scale in correlator output units

Flux scaling

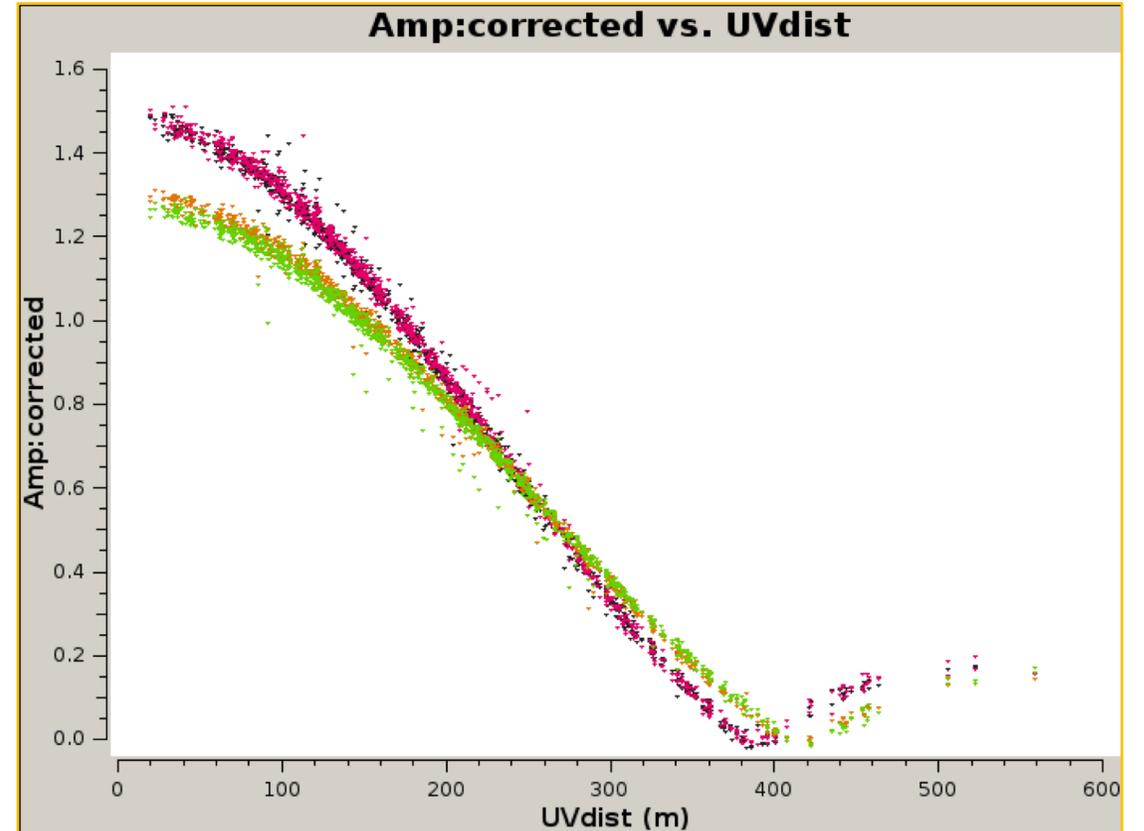
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CASA model of Titan for a given dataset
(Notice the y-axis scale)

Flux scaling

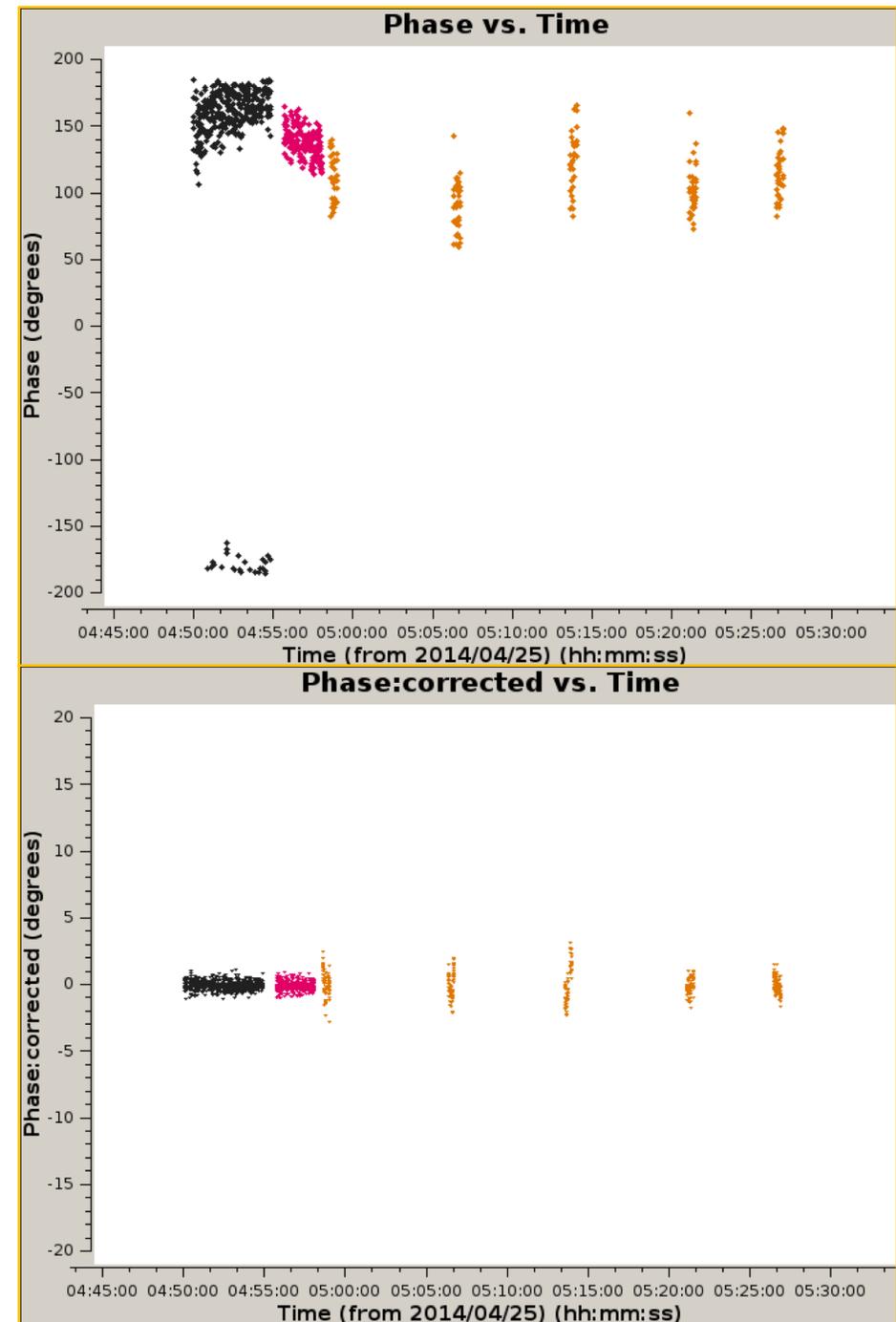
- This is typically a two part process, where by you set your amplitude scales with one task (e.g. setJy in CASA) and then scale the fluxes of other sources with another (e.g. fluxscale in CASA).



Fully calibrated data for Titan

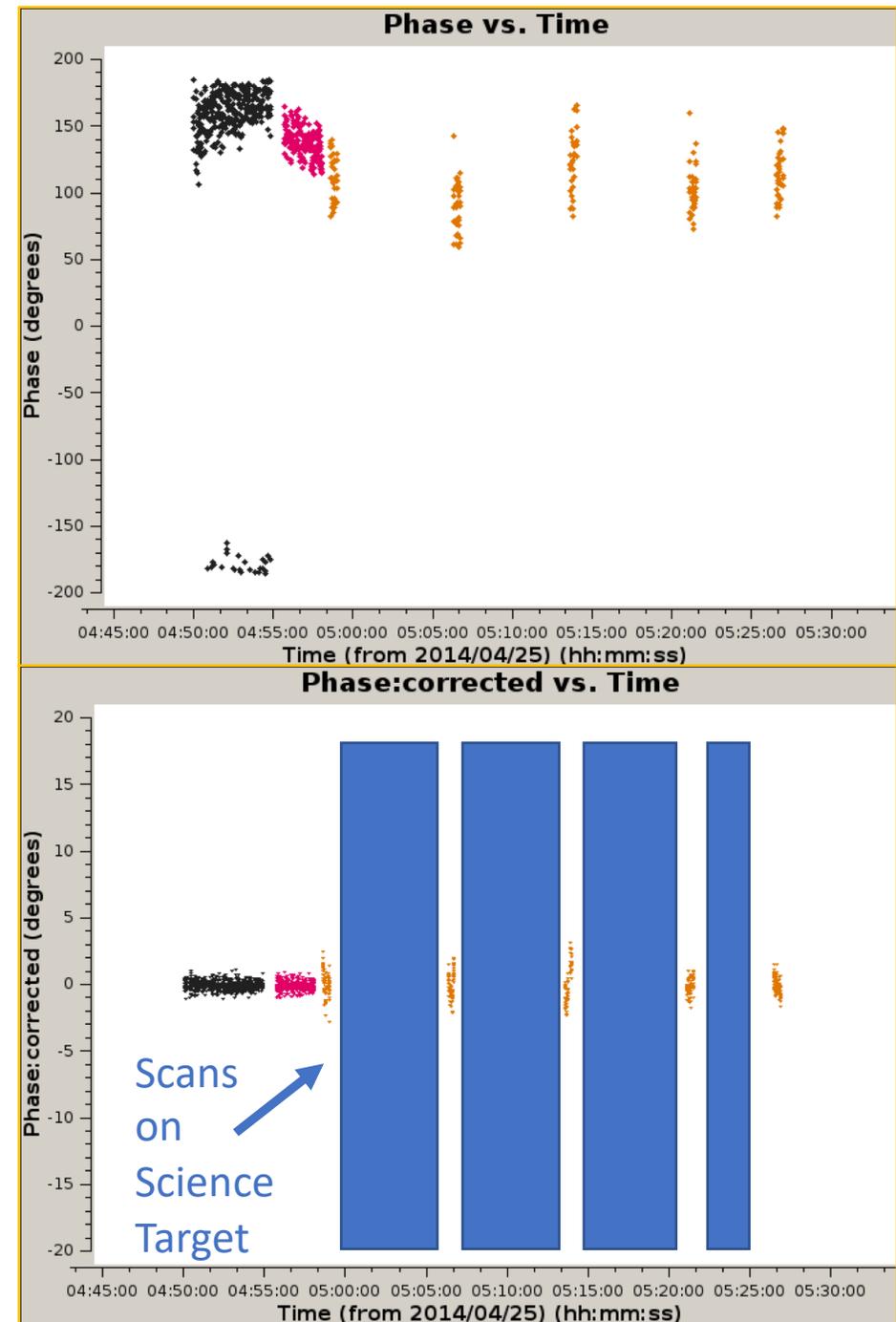
Complex Gain (*Phase and Amplitude*)

- Calibration step to mitigate temporal effects (i.e. the atmosphere) on the phase and amplitude of the target source visibilities.
- Target a bright, *nearby* (few degrees) quasar and intersperse throughout observation of target.
- Needs to be nearby to be experiencing the same atmospheric conditions



Complex Gain (Phase and Amplitude)

- Calibration step to mitigate temporal effects (i.e. the atmosphere) on the phase and amplitude of the target source visibilities.
- Target a bright, *nearby* (few degrees) quasar and intersperse throughout observation of target.
- Needs to be nearby to be experiencing the same atmospheric conditions



Notice y-axis is factor 10 smaller!

Imaging and cleaning

What we measure (reminder)

Correlator output

$$R_{x,y} = A_0 |V| \Delta \nu \cos(2\pi \nu \mathbf{b}_\lambda \cdot \mathbf{s}_0 - \phi_V)$$

Eq. 1

Visibility equation

$$V(u, v) = \int A(l, m) I(l, m) e^{-i2\pi(ul+vm)} \frac{dldm}{\sqrt{1-l^2-m^2}}$$

Eq. 2

Dirty image

$$I^D(l, m) = \iint_{-\infty}^{\infty} S(u, v) V(u, v) e^{2\pi i(ul+vm)} dudv$$

Eq. 3

Sampling function

Let us call the sampling of the uv -plane (aka uv -coverage), \mathcal{S} , the sampling function

$$S(u, v) = \sum_{k=1}^M \delta(u - u_k, v - v_k)$$

Given this the synthesized beam, B , is $B = \mathbf{FT}(S)$.

And for each uv -point we have an observed visibility, $V(u, v)$, so we can define the *sampled visibility function* as:

$$V^S(u, v) \equiv \sum_{k=1}^M \delta(u - u_k, v - v_k) V(u_k, v_k)$$

So $V^S = SV$ and from earlier (eq. 3) $I^D = \mathbf{FT}(V^S) = \mathbf{FT}(SV)$. From which it follows that I^D is the measured sky brightness convolved with the synthesis beam, B .

Weighting

Given this we can introduce weighting functions to control the shape of the synthesised beam.

DON'T WORRY TOO MUCH ABOUT THIS NOW, IT IS A SUBTLETY YOU NEED TO THINK ABOUT WHEN ACTUALLY IMAGING...

$$W(u, v) = \sum_{k=1}^M R_k D_k T_k \delta(u - u_k, v - v_k)$$

R_k = Weights relating to data quality, i.e. down weight bad data. This is observation dependent and we have no post observation control over it (so ignore).

T_k = Tapering function. Apply a tapering function (i.e. Gaussian), to the uv -coverage to for example downweight the outer uv -points lowering resolution.

D_k = Density weighting, applies some weight based on the clustering of uv points on a grid... This is the most commonly used type of weighting.

And as per the previous slide we can define the *weighted and sampled visibility function* as $V^W = WV$.

Imaging the data

We now have the sampled and weighted visibilities, V^W .

In order to efficiently make an image of our target sky brightness distribution, I , we need to take the Fourier transform of this using Fast Fourier Transforms (FFTs).

This requires the V^W data to be gridded on to a regular grid.

This is done by convolution with some suitable gridding function*. Leaving us ultimately with some weighted and gridded visibilities which can be FFT'd to give us our dirty image P .

*Discussion of gridding algorithms is a little beyond the scope of this workshop, please check the references at the start of this talk for more information.

CLEAN-ing

We've seen that our dirty image \mathcal{I}^D is the sky brightness distribution convolved with the synthesised beam \mathcal{B} .

To get a better representation of the sky brightness distribution we need to remove the artefacts introduced by \mathcal{B} . To achieve this we use CLEAN*

*As far as I know CLEAN is not an acronym, but it is usually capitalised. I don't know why?!

Simple CLEAN overview

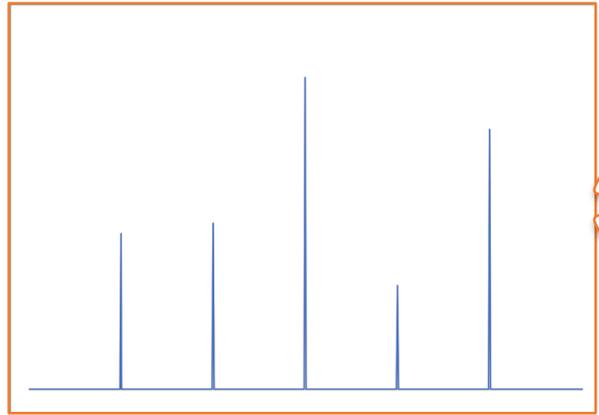
The Högbom (1974) CLEANing algorithm is the simplest CLEAN algorithm and is very illustrative of how CLEAN works in general.

In words the Högbom algorithm works as follows:

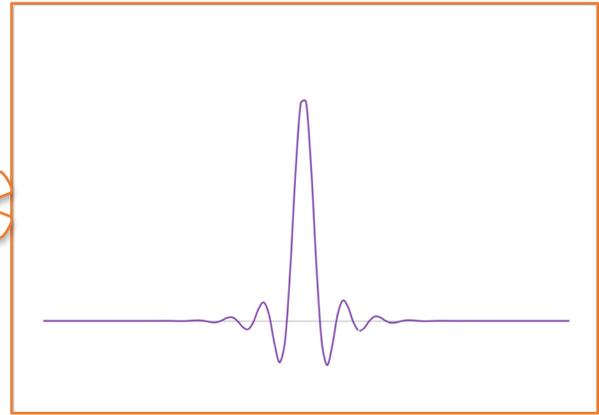
- 1) Find the magnitude and position of peak emission in the dirty image.
- 2) Subtract from the dirty image the dirty beam, B , scaled by some gain value (i.e. 0.1). Creating a 'residual' image.
- 3) Note the position and magnitude subtracted as a point in a model.
- 4) Repeat 1-3 until a user defined threshold is reached, either some noise limit (in the residual) or a given number of iterations.
- 5) Convolve the final model with an idealised beam. I.e. a beam based on the interferometer if it was a huge single dish.

Or in a 2D example on the next page

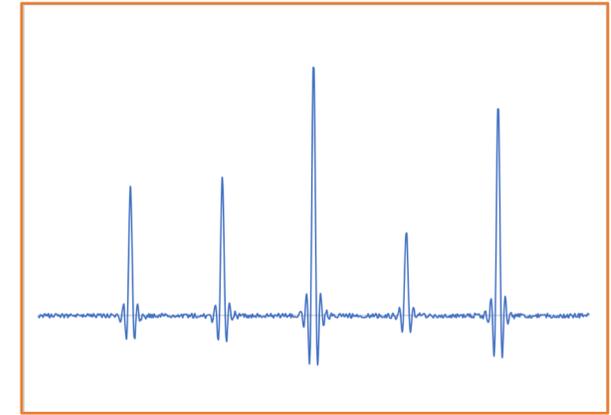
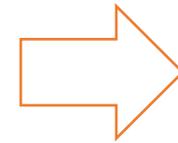
Högbom 2D example



True sky of 5 point sources

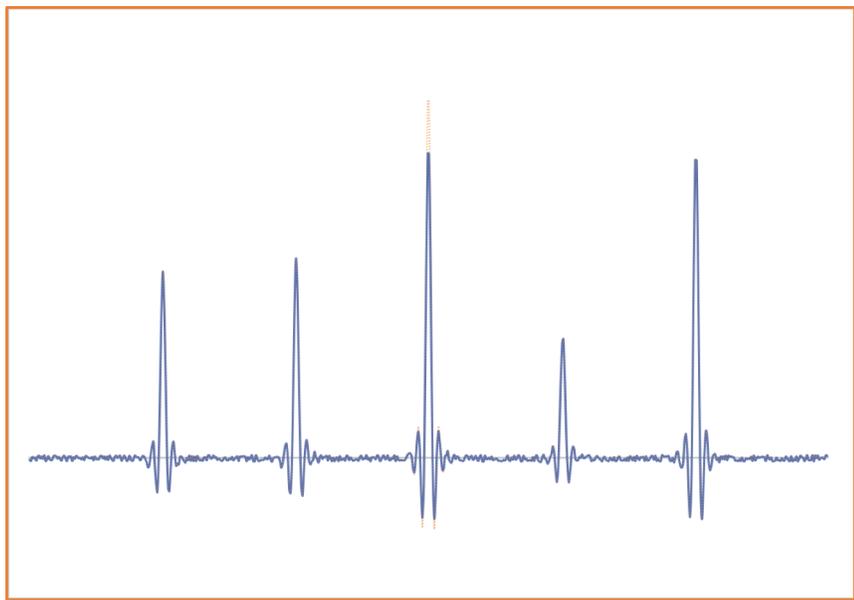


Dirty Beam (DB)

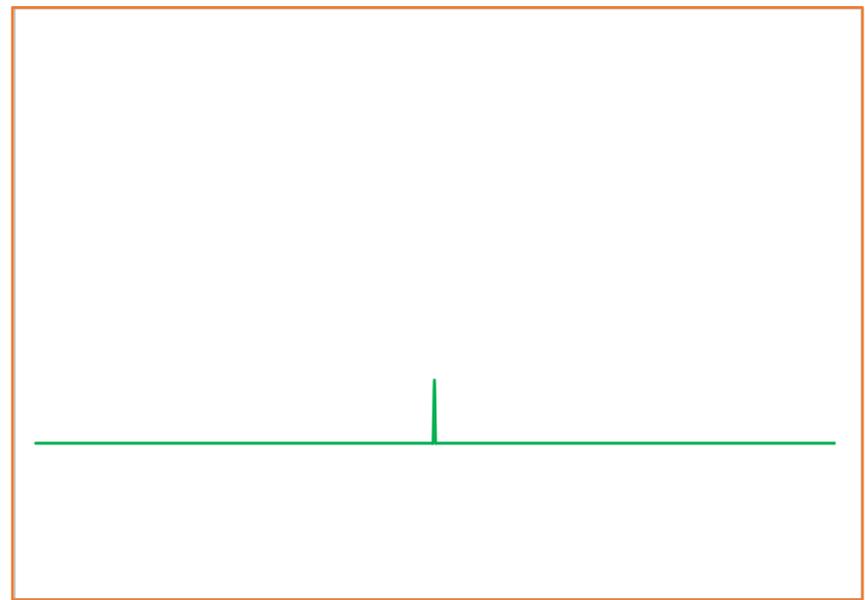


Dirty Image + noise, (DI)

Iteration 1

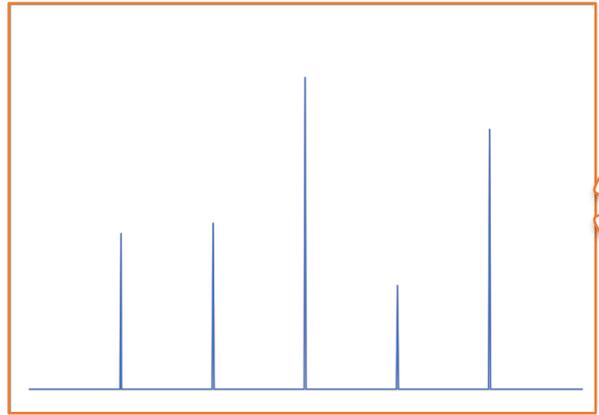


Residual Image after subtracting DB from peak in DI

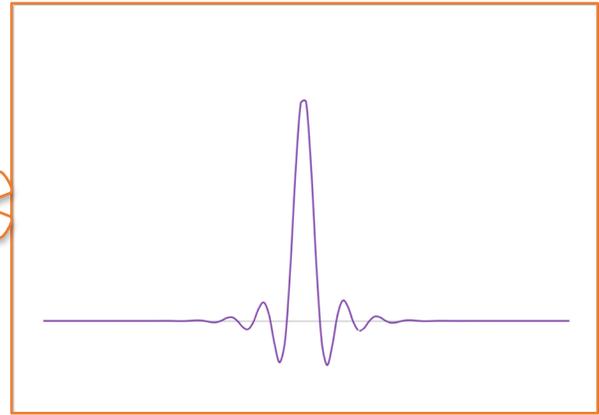


CLEAN components in Model Image

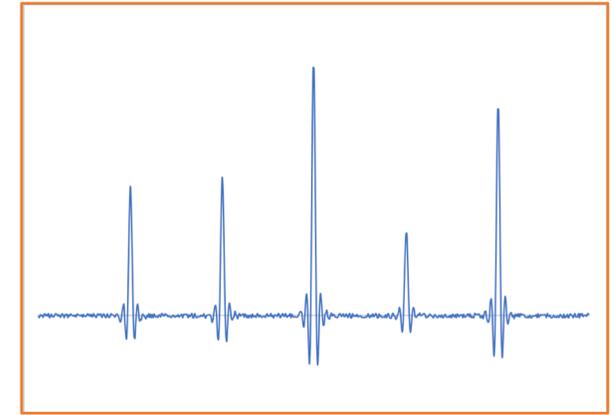
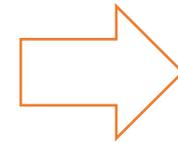
Högbom 2D example



True sky of 5 point sources

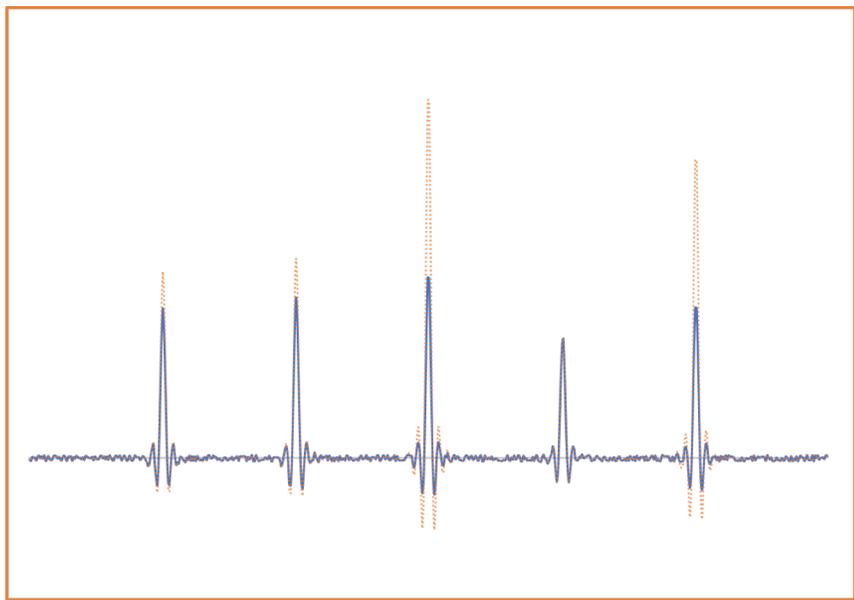


Dirty Beam (DB)

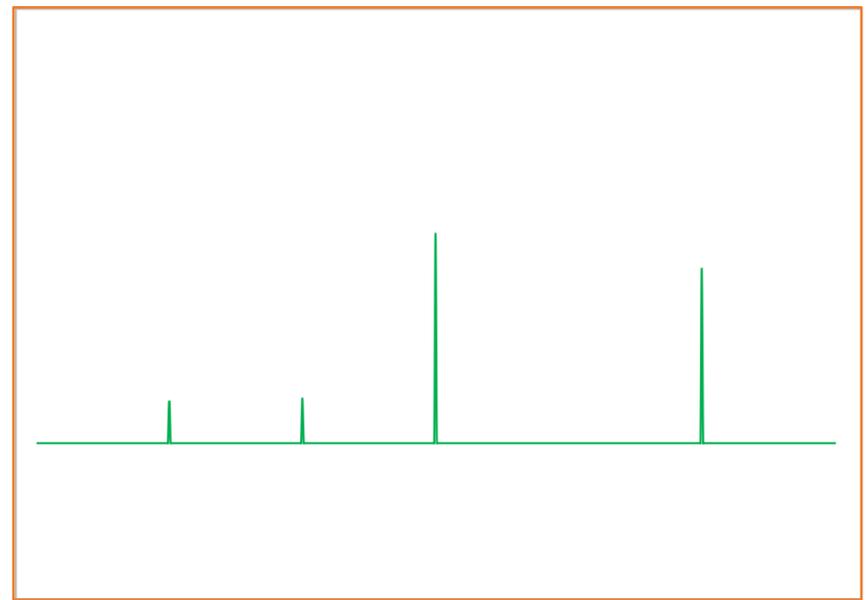


Dirty Image + noise, (DI)

Iteration n

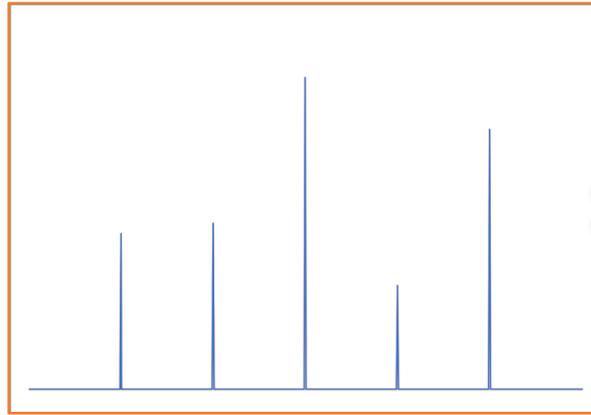


Residual Image after subtracting several DBs from located peaks in DI

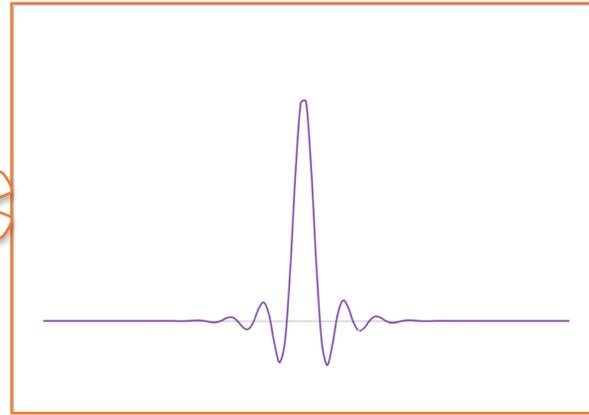


CLEAN components in Model Image

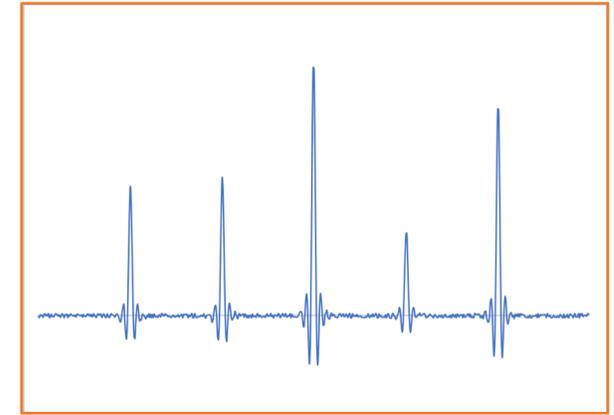
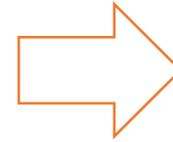
Högbom 2D example



True sky of 5 point sources

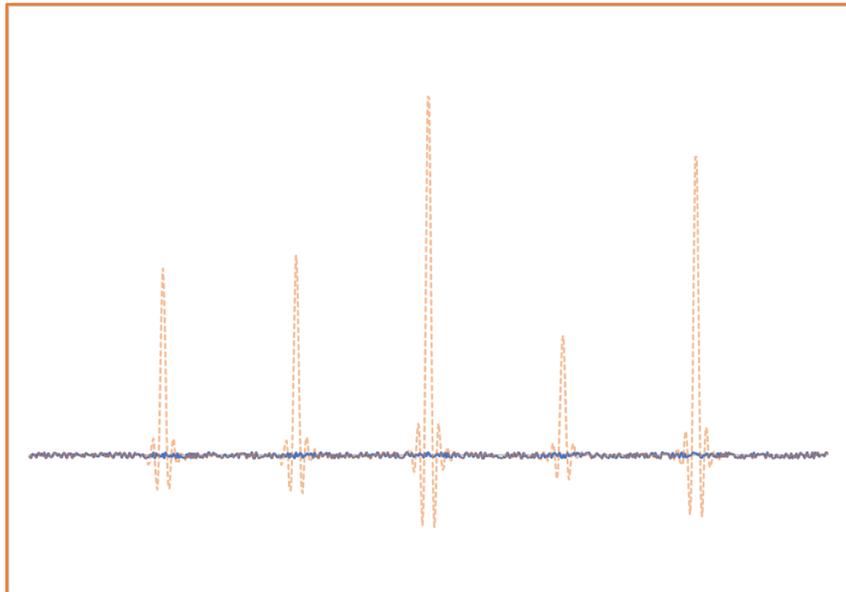


Dirty Beam (DB)

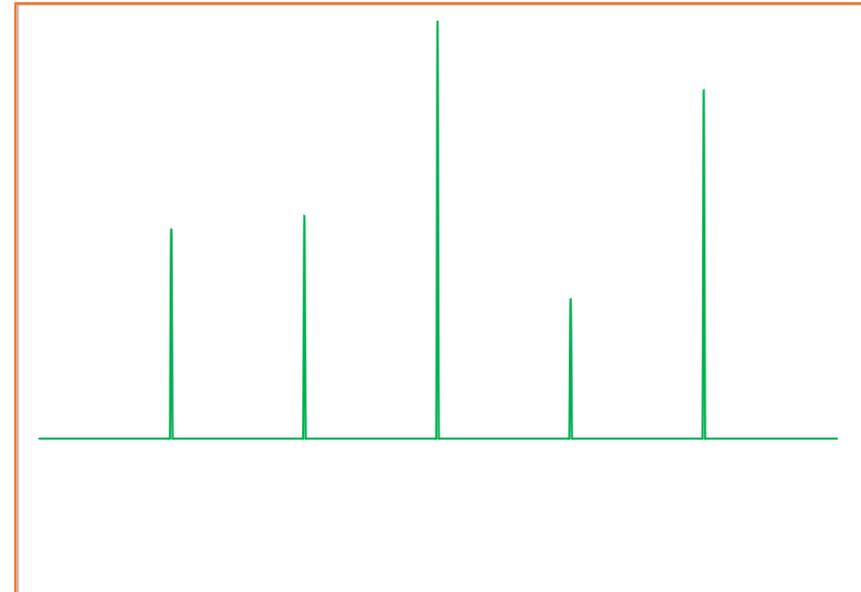


Dirty Image + noise, (DI)

Iteration *FINAL*

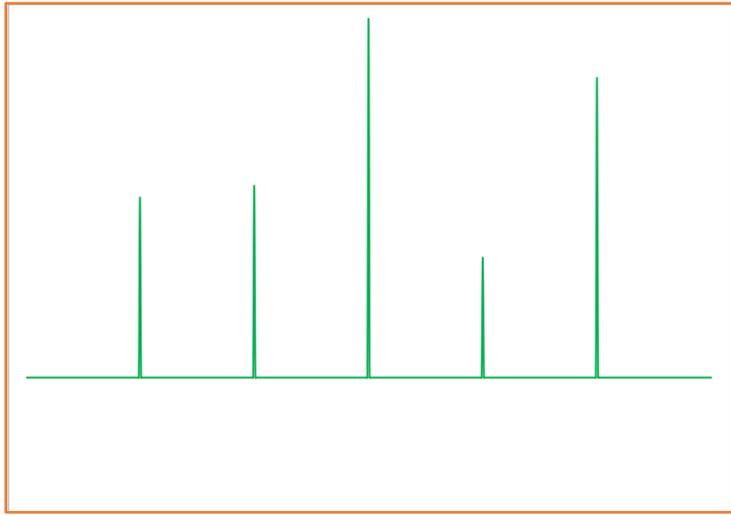


Residual Image after subtracting enough DBs from located peaks in DI until threshold met.

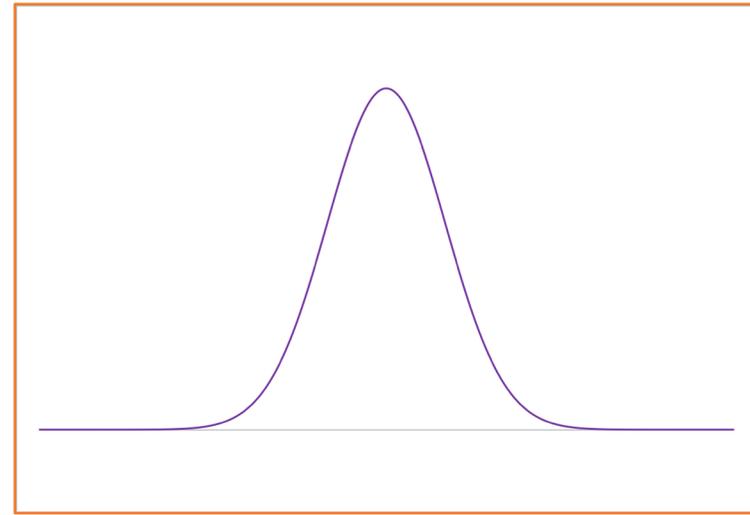
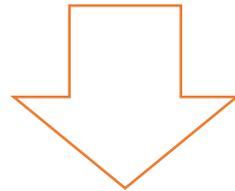


CLEAN components in model Image after final CLEAN loop

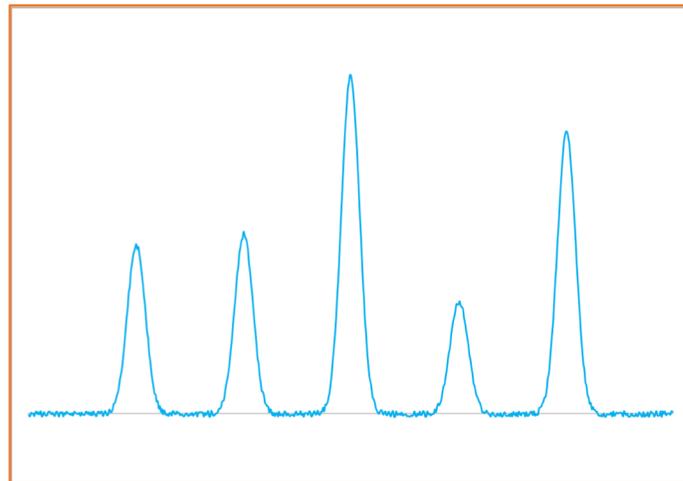
Högbom 2D example



CLEAN components in model Image after final CLEAN loop

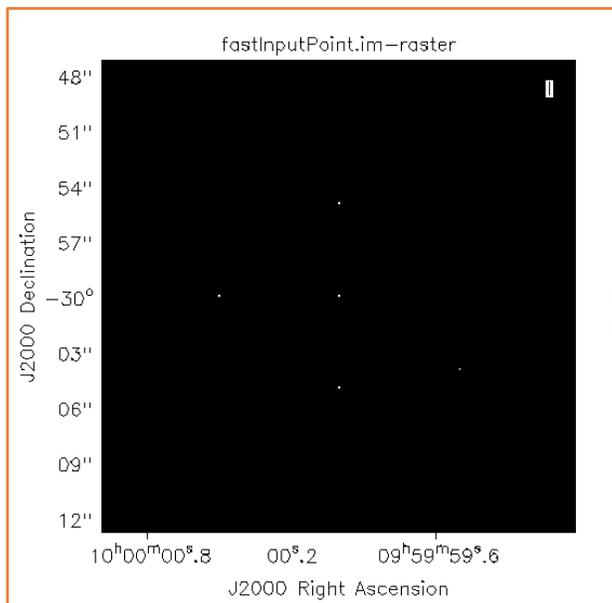


Idealized beam

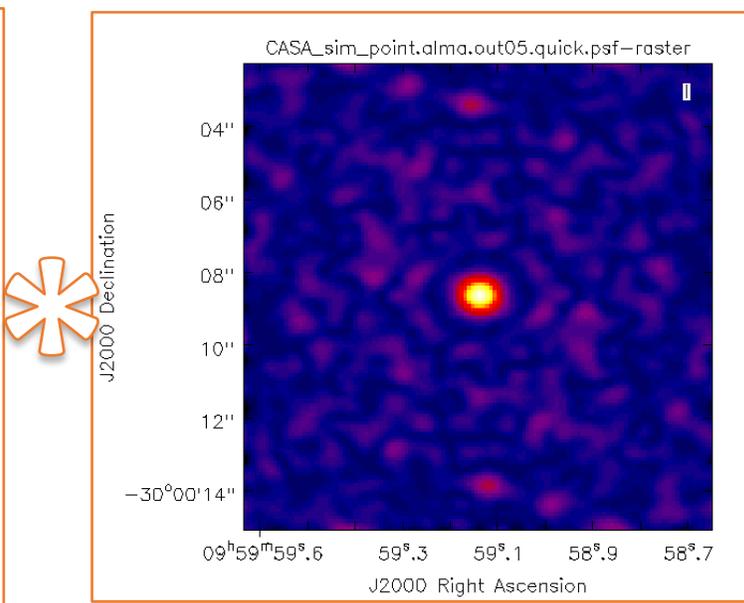


Final reconstructed image

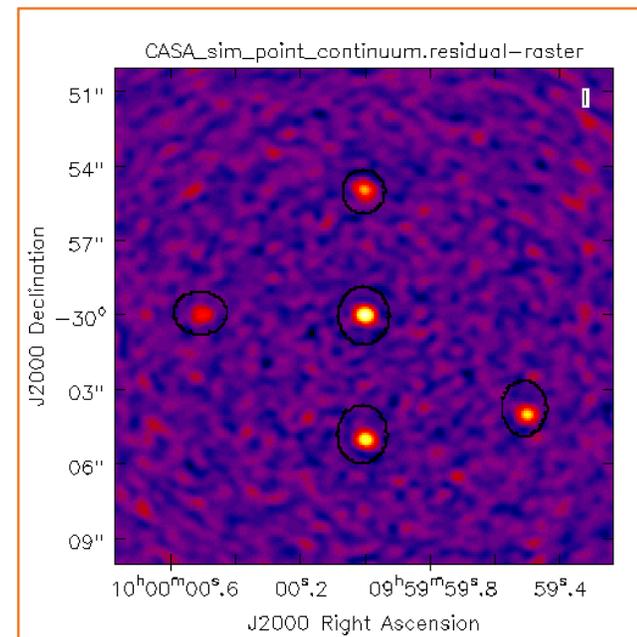
... and in 3D



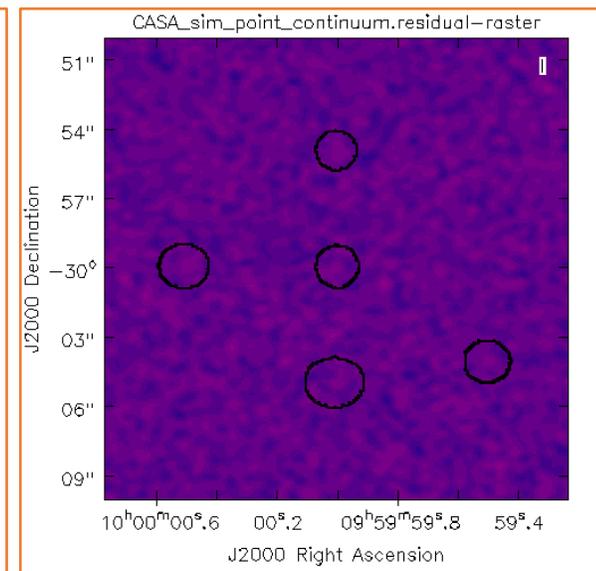
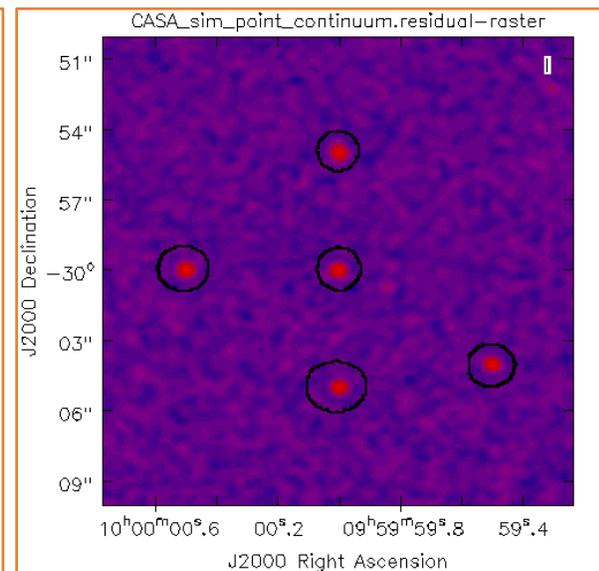
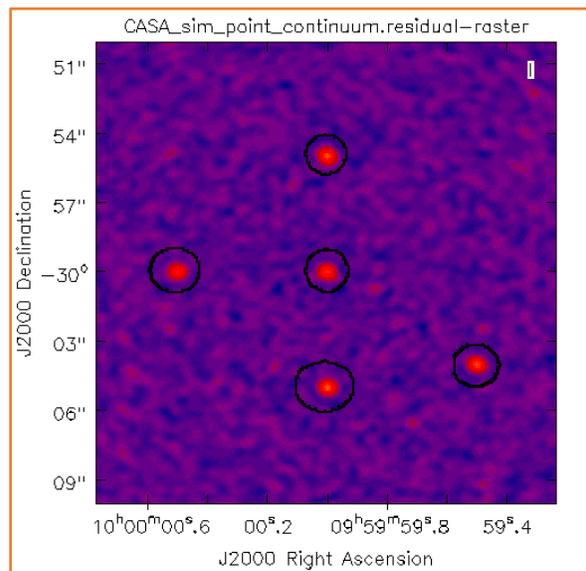
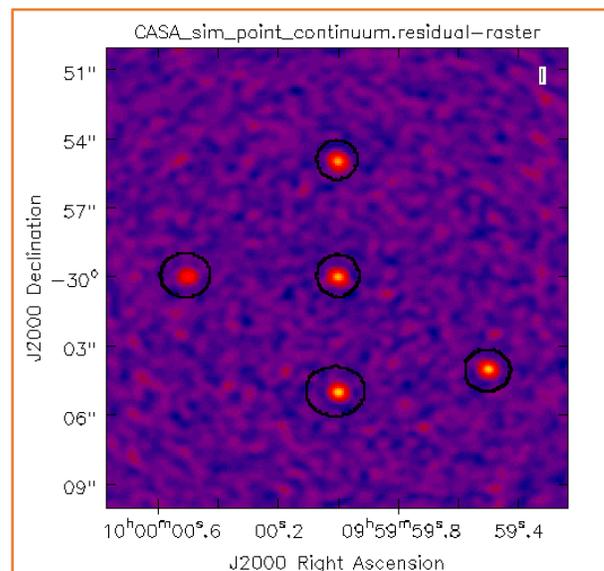
True sky of 5 point sources



Dirty Beam (DB)

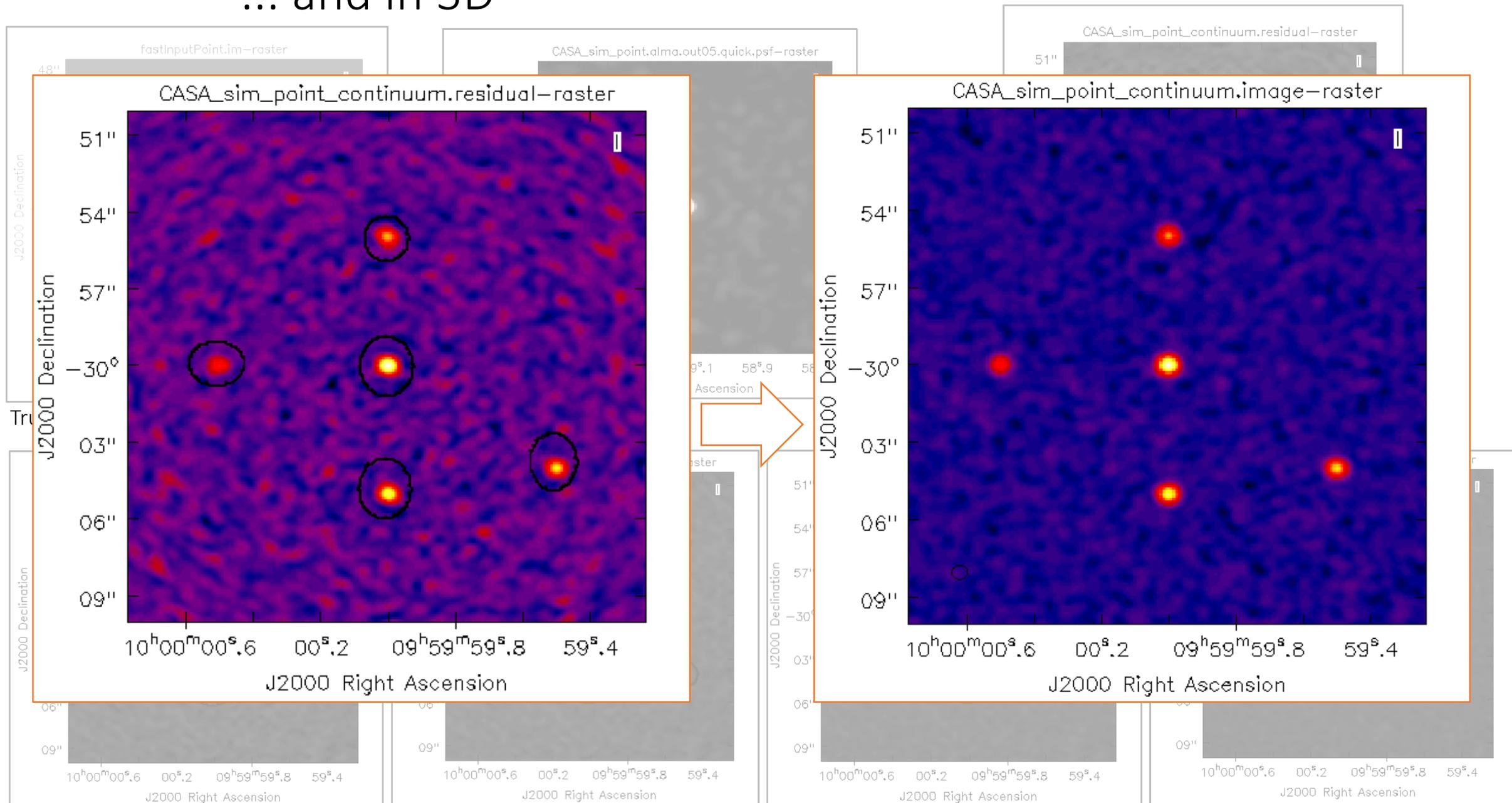


Dirty Image + noise, (DI)



→ Increasing CLEAN cycles →

... and in 3D



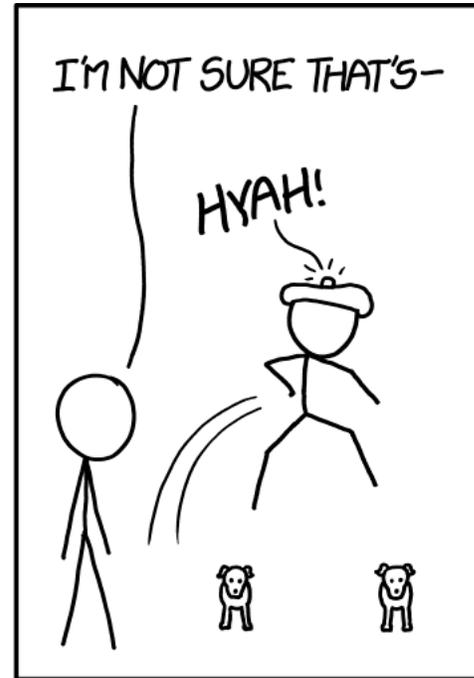
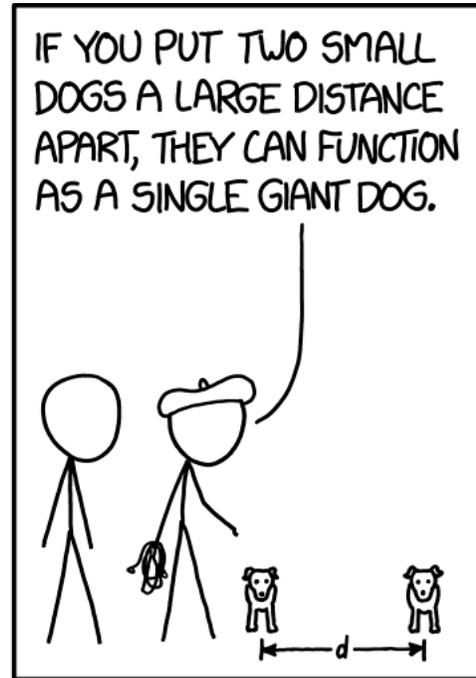
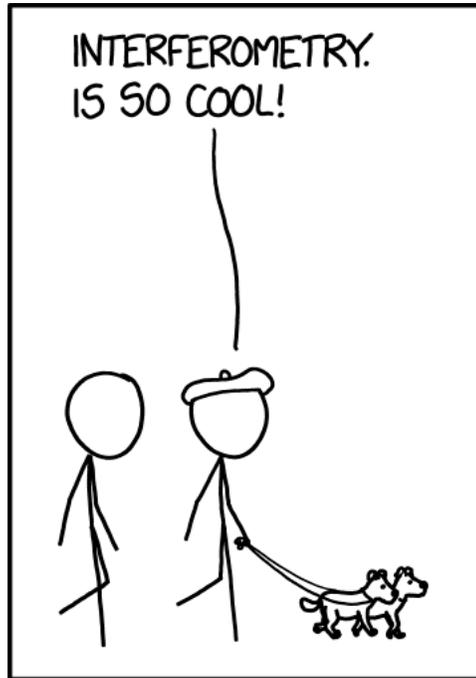
And after all that you'll have a nice image/data cube from which you can actually do some Science!

Further Reading

The slides from this talk are based on the fundamentals of interferometry which are explained in detail across:

- *“Interferometry and Synthesis in Radio Astronomy”* - Thompson, Moran & Swenson
- *“Synthesis Imaging in Radio Astronomy II”* – NRAO
- *“An introduction to Radio Astronomy”* – Burke and Graham-Smith (4th edition out soon Burke, Graham-Smith (& Wilkinson?))
- *“Tools of Radio Astronomy”* – Wilson, Rohfls & Hüttemeister
- *“The CASA Cookbook”* – Ott & Kern et al.

¡Cheers!



¿Questions?