MANCHESTER 1824

The University of Manchester

Interferometer fundamentals

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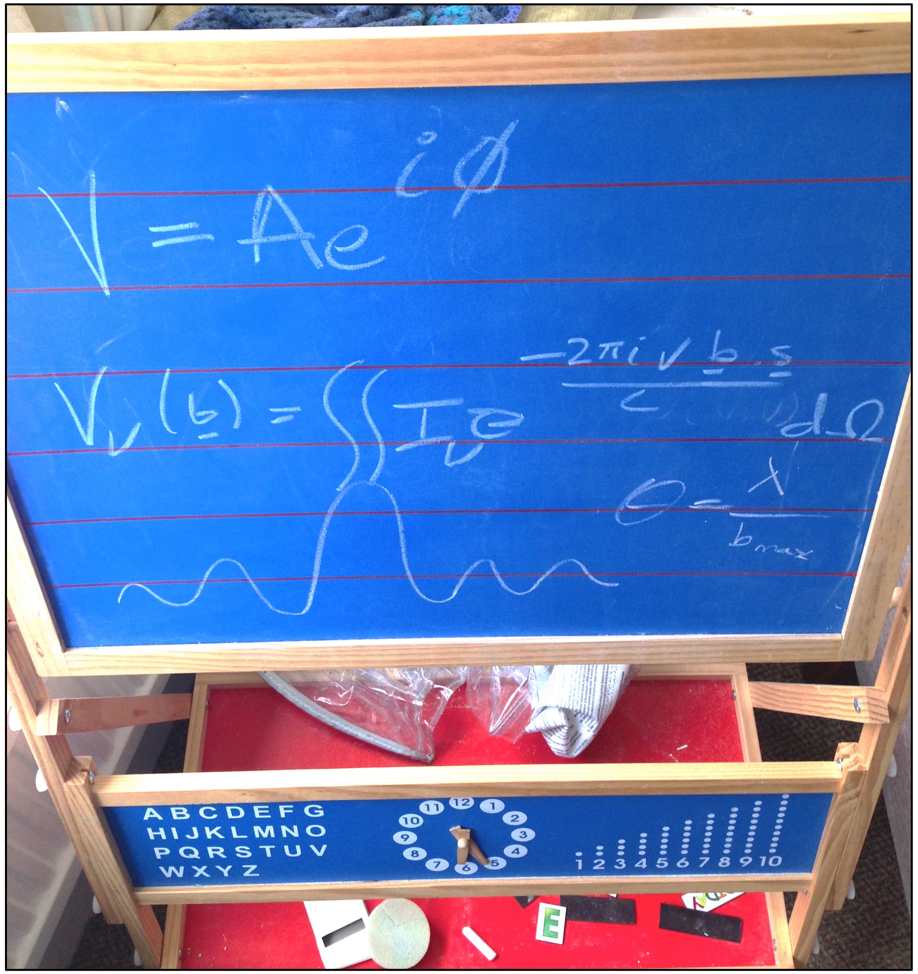
Outline

- Why use an interferometer?
- Two element interferometer
- *uv*-coverage
- Imaging data and Cleaning



Why use an interferometer?
- What do astronomers want?

- High resolution

$$\theta \approx \frac{\lambda}{D}$$

- High sensitivity

$$S_{\nu,\,rms} = \frac{2kT_{sys}}{A_e\sqrt{\tau\,\Delta\nu}}$$

Resolution

Imagine we want Hubble resolution:

D=2.4m, λ=600nm -> θ=0.05arcsec

At radio wavelengths, say 4.5cm (6.7GHz):

D would need to be ~185km

Sensitivity

Basically, the bigger the better!

$$S_{\nu,\, rms} = \frac{2kT_{sys}}{A_e \sqrt{\tau\, \Delta\nu}}$$

Effective area of your dish

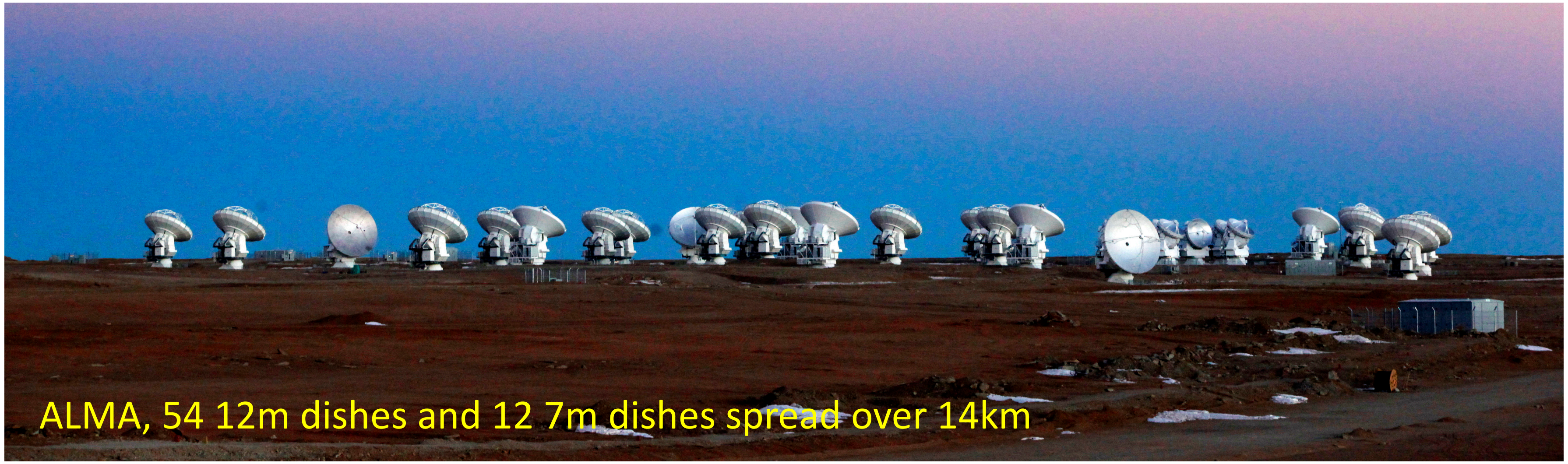
How do we get it?

- Interferometry

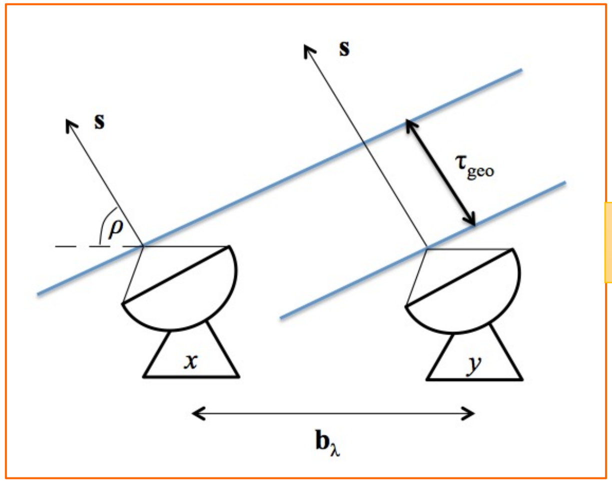
$$\theta \approx \frac{\lambda}{b_{\max}}$$

$$\Delta S_{rms} = \frac{2kT_{sys}}{A_e\sqrt{N(N-1)\tau\,\Delta\nu}}$$

$$MRS \sim 0.6\,\frac{\lambda}{b_{min}}$$

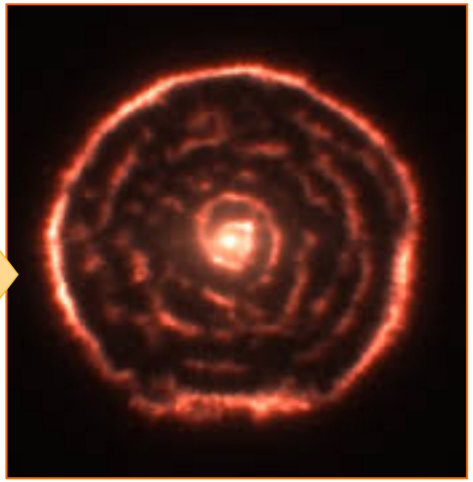


Most peoples first impression of interferometry



Dishes

Dark Arts

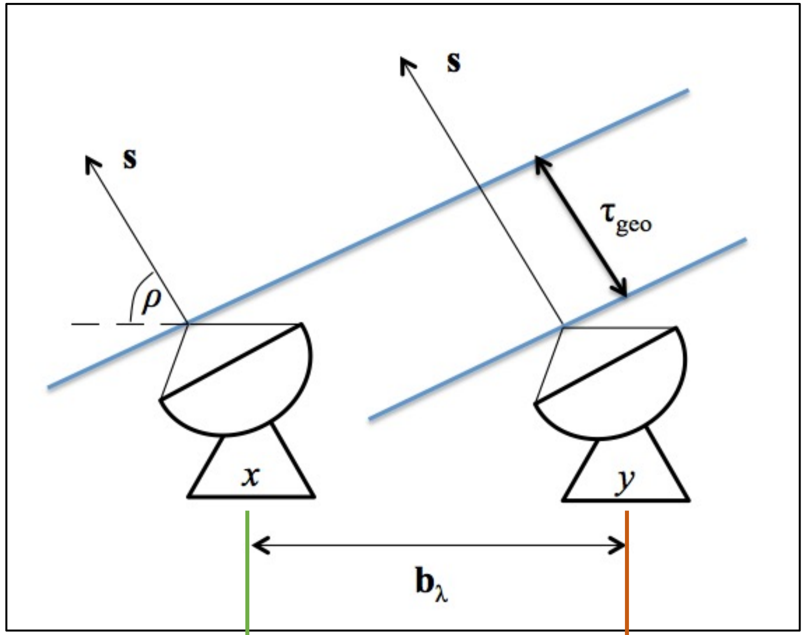


Pretty pictures

... hopefully by the end of this talk we'll have cleared up the bit in the middle

The two element interferometer

The two element interferometer



What are we trying to do?

- Recreate the sky brightness distribution of astrophysical sources.

Delay between wavefronts arriving at x then y:

$$\tau_{geo} = \frac{\mathbf{b}\cdot\mathbf{s}}{c} = \frac{bs\cos\rho}{c}$$

$$x(t) = \mathrm{v}_1 \cos 2\pi\nu t \qquad y(t) = \mathrm{v}_2 \cos 2\nu\pi(t+\tau_{geo})$$

Receiver outputs

$$R_{x,y}(\tau_{geo}) = x \otimes y = X(\nu)Y^*(\nu)$$

Correlator's function

Do the maths

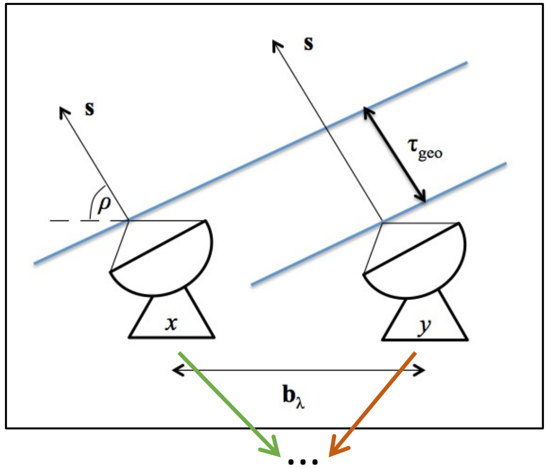
$$R_{x,y}(\tau_{geo}) = \mathrm{v}_1 \mathrm{v}_2 \cos 2\pi\tau_{geo}$$

Correlator output

That's all well and good but how does this

$$R_{x,y}(\tau_{geo}) = \mathrm{v}_1 \mathrm{v}_2 \cos 2\pi\nu\tau_{geo}$$

tell us anything about the sky brightness distribution?



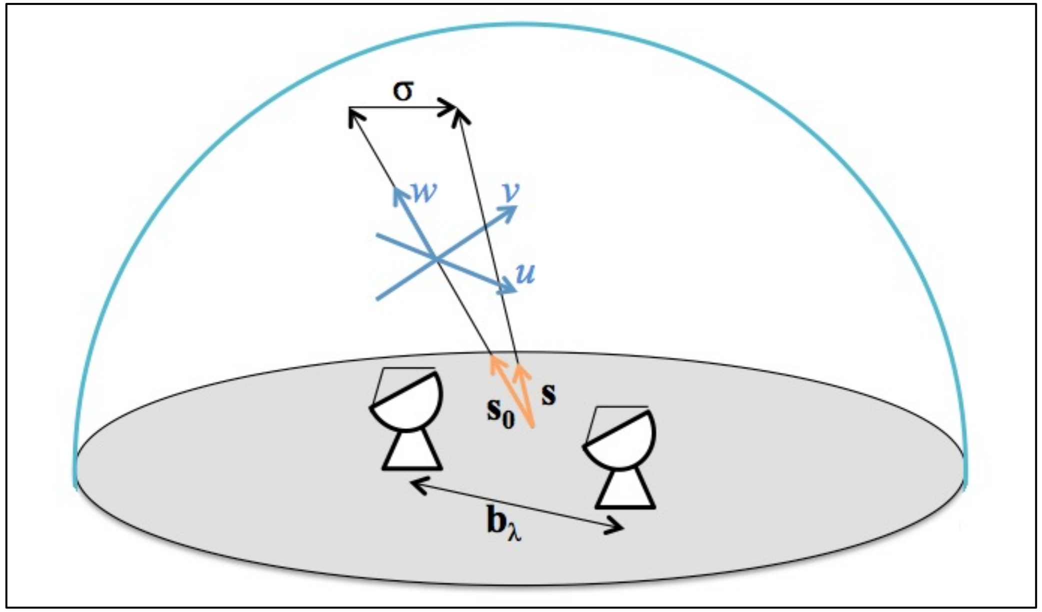
$$R_{x,y}(\tau_{geo}) = v_1 v_2 \cos 2\pi\nu\tau_{geo}$$

v_1 and v_2, the voltage outputs of x & y are directly related to:

- The brightness distribution, $I(\mathbf{s})$, of the astronomical object
- as seen over solid angle $d\Omega$
- and $A(\mathbf{s})$ the area of the dish we use to observe it.

Leading to ...

$$R_{x,y}(\tau_{geo}) = \Delta\nu \int A(\mathbf{s}) I(\mathbf{s}) \cos 2\pi \mathbf{b}_\lambda \cdot \mathbf{s} \; d\Omega$$



Adding in a bit more reality...

- The vector $\mathbf{s}$ is comprise of the addition of $\mathbf{s_0}$ and σ (so $\mathbf{s}=\mathbf{s_0}+\sigma$).
- We set τ_{geo} to zero with instrumental delays
- Meaning all delays in the data are from the vector σ

We then define the Complex Visibility as:

$$V \equiv |V| e^{i\phi_V} = \int A(\sigma) I(\sigma)\, e^{-i2\pi \mathbf{b}_\lambda \cdot \sigma} \mathrm{d}\Omega$$

which is rather nice as V is the Fourier transform of I.

Relating the visibility equation to the correlator output gives

$$R_{x,y}(\tau_{geo}) = \Delta\nu \int A(\mathbf{s}) I(\mathbf{s}) \cos 2\pi \mathbf{b}_\lambda \cdot \mathbf{s} \, \mathrm{d}\Omega$$

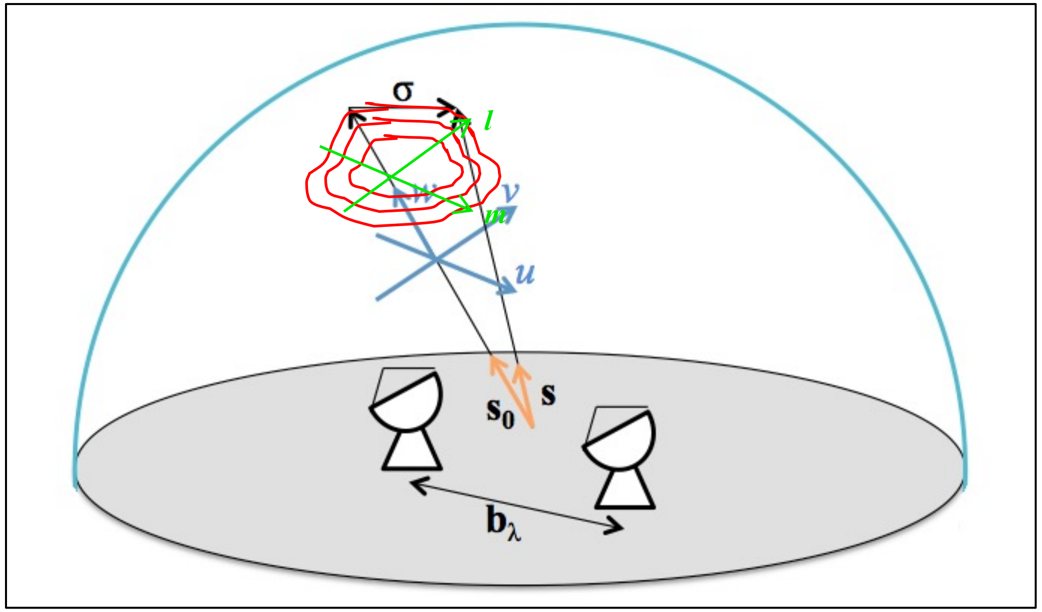
$$V \equiv |V| e^{i\phi_V} = \int A(\sigma) I(\sigma)\, e^{-i2\pi \mathbf{b}_\lambda \cdot \sigma} \mathrm{d}\Omega$$

$$R_{x,y} = A_0 |V| \Delta\nu \cos(2\pi\nu \mathbf{b}_\lambda \cdot \mathbf{s}_0 - \phi_V)$$

Known!*

* After proper calibration

A coordinate system for interferometry



We define u and v, as E-W and N-S positions w.r.t w axis which is parallel to s_0.

l and m as direction cosines of $\mathbf{s}$ we can write the visibility equation as:

$$V(u,v) = \int A(l,m) I(l,m)\, e^{-i2\pi(ul+vm)} \frac{\mathrm{d}l\mathrm{d}m}{\sqrt{1-l^2-m^2}}$$

Given l and m are small the small angle approx applies and $V(u,v)$ becomes a direct Fourier transform of $I(x,y)$

In its full gory glory the... Measurement Equation

$$V_{ij} = M_{ij}B_{ij}G_{ij}D_{ij}\int E_{ij}P_{ij}T_{ij}F_{ij}SI_{\nu}(l,m)\,e^{-i2\pi(u_{ij}l+v_{ij}m)}\frac{\mathrm{d}l\mathrm{d}m}{\sqrt{1-l^2-m^2}} + Q_{ij}$$

V_{ij} = What we measure
I_{ν} = What we want
Q_{ij} = additive errors
S = maps I to polarisation
i,j = telescope pair
M_{ij} = Multiplicative baselines errors
B_{ij} = Bandpass reponse
G_{ij} = Gerenalised electronic gain
D_{ij} = polarisation leakage
E_{ij} = Antenna voltage pattern
P_{ij} = paralatic angle
T_{ij} =Tropospheric effects

Green= vectors
Blue= Scalars
Red= Part of the Jones Matrix

Slide cribbed from A.M.S.Richards

What we measure

Correlator output

$$R_{x,y} = A_0 |V| \Delta \nu \cos(2\pi \nu \mathbf{b}_\lambda \cdot \mathbf{s}_0 - \phi_V)$$

Eq. 1

Visibility equation

$$V(u,v) = \int A(l,m) I(l,m)\, e^{-i2\pi(ul+vm)} \frac{dl dm}{\sqrt{1-l^2-m^2}}$$

Eq. 2

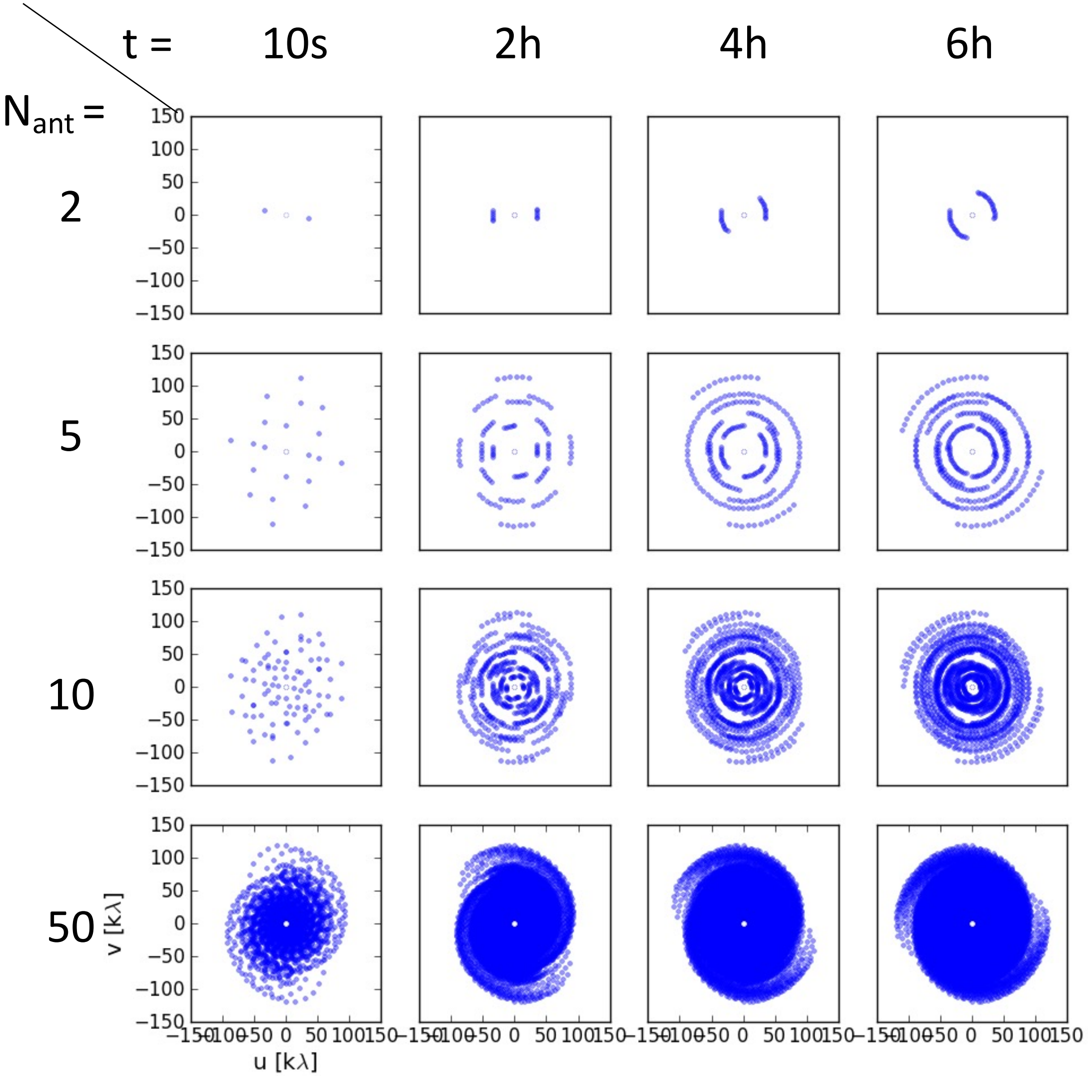
Dirty image

$$I^D(l,m) = \iint_{-\infty}^{\infty} S(u,v) V(u,v) e^{2\pi i(ul+vm)} du dv$$

Eq. 3

uv-coverage

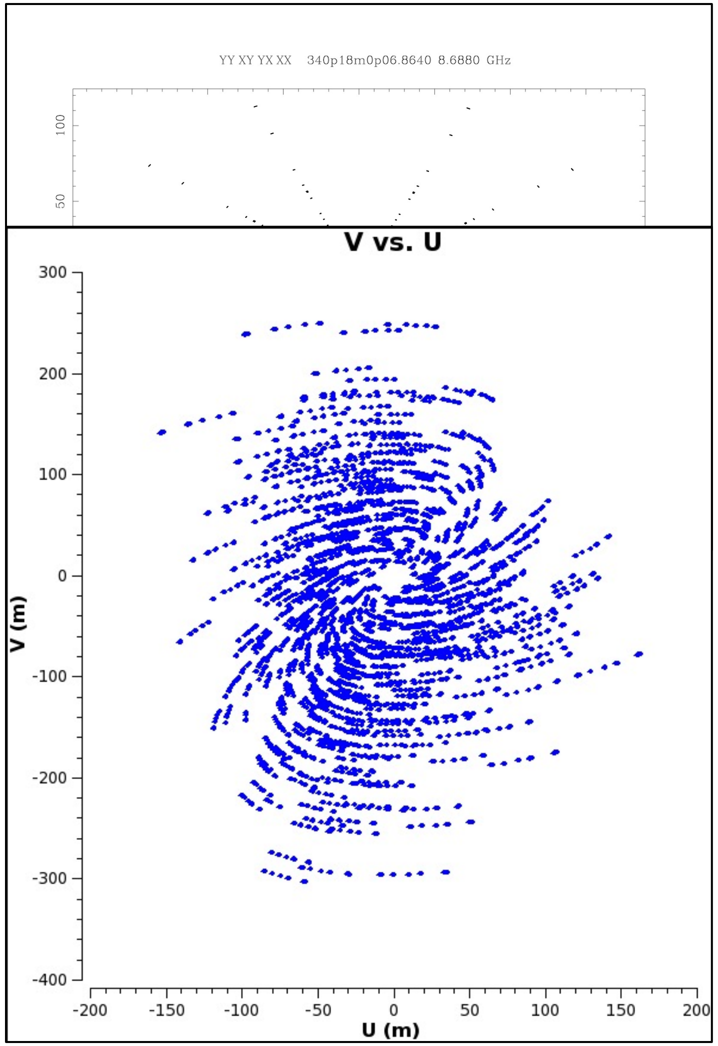
Filling the *uv*-plane



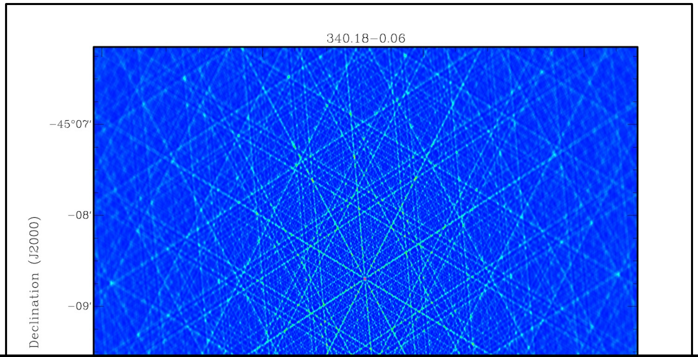
uv-coverage of an interferometer set out in a logarithmic spiral pattern comprised of two, five, ten and fifty antennas (top to bottom)
and observing for 10 s, 2, 4, and 6 h (left to right).

Filling the *uv*-plane

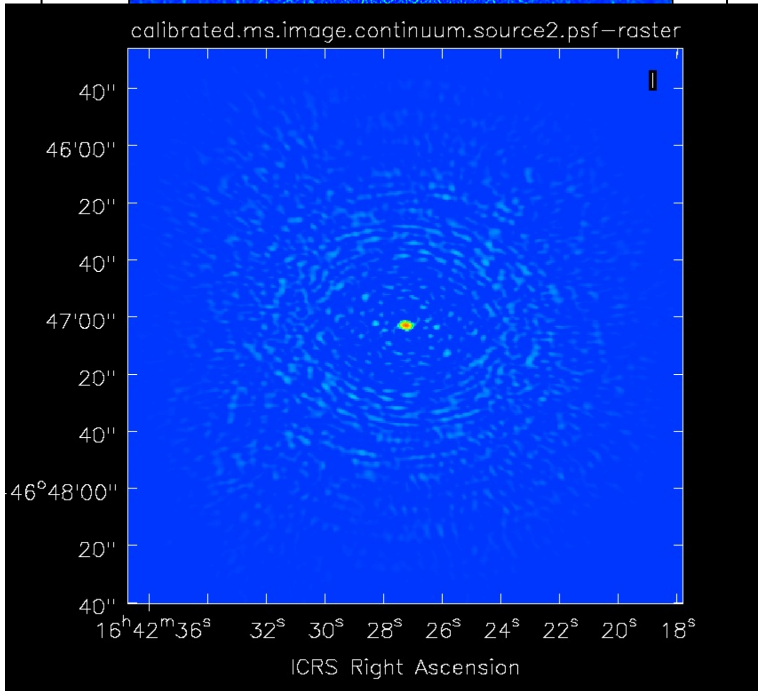
We want to fill the *uv*-plane because the *uv*-coverage is the FT of the synthesised beam, *B*. The greater the *uv*-coverage the better behaved the sidelobes are.



6 dishes
(~15 mins)



42 dishes
(~15 mins)



Filling the *uv*-plane

$$V(u,v) = \int A(l,m) I(l,m)\, e^{-i2\pi(ul+vm)} \frac{\mathrm{d}l\mathrm{d}m}{\sqrt{1-l^2-m^2}}$$

For each antenna pair at each integration interval we get one *uv* measurement.

To optimize *uv*-coverage, thus giving us a ‘nicer’ synthesized beam we would ideally have:

1) A large number of dishes, thus more antenna pairs (N(N-1))
2) Greater time on source, more *uv*-points from the antennas we have.
3) An array configuration with a larger number of unique antenna spacings.

Visual Example

Calibration will be covered tomorrow

Imaging and cleaning

What we measure (reminder)

Correlator output

$$R_{x,y} = A_0 |V| \Delta\nu \cos(2\pi\nu \mathbf{b}_\lambda \cdot \mathbf{s}_0 - \phi_V)$$

Eq. 1

Visibility equation

$$V(u,v) = \int A(l,m) I(l,m)\, e^{-i2\pi(ul+vm)} \frac{dl dm}{\sqrt{1-l^2-m^2}}$$

Eq. 2

Dirty image

$$I^D(l,m) = \iint_{-\infty}^{\infty} S(u,v) V(u,v) e^{2\pi i(ul+vm)} du dv$$

Eq. 3

Sampling function

Let us call the sampling of the uv-plane (aka uv-coverage), S, the sampling function

$$S(u, v) = \sum_{k=1}^{M} \delta(u - u_k, v - v_k)$$

Given this the synthesized beam, B, is $B = \mathbf{FT}(S)$.

And for each uv-point we have an observed visibility, $V(u,v)$, so we can define the *sampled visibility function* as:

$$V^S(u, v) \equiv \sum_{k=1}^{M} \delta(u - u_k, v - v_k)\, V(u_k, v_k)$$

So $V^s = SV$ and from earlier (eq. 3) $I^D = \mathbf{FT}(V^S) = \mathbf{FT}\,(SV)$. From which it follows that I^D is the measured sky brightness convolved with the synthesis beam, B.

This is a truncated version of Chapter 7 (Briggs, Schwab & Sramek) of *"Synthesis Imaging in Radio Astronomy II"*

Weighting

Given this we can introduce weighting functions to control the shape of the synthesised beam.

DON'T WORRY TOO MUCH ABOUT THIS NOW, IT IS A SUBTLETY YOU NEED TO THINK ABOUT WHEN ACTUALLY IMAGING...

$$W(u,v) = \sum_{k=1}^{M} R_k D_k T_k \delta(u - u_k, v - v_k)$$

$\boldsymbol{R_k}$= Weights relating to data quality, i.e. down weight bad data. This is observation dependent and we have no post observation control over it (so ignore).

$\boldsymbol{T_k}$= Tapering function. Apply a tapering function (i.e. Gaussian), to the uv-coverage to for example downweight the outer uv-points lowering resolution.

$\boldsymbol{D_k}$= Density weighting, applies some weight based on the clustering of uv points on a grid... This is the most commonly used type of weighting.

And as per the previous slide we can define the *weighted and sampled visibility function* as $V^W = WV$.

This is a truncated version of Chapter 7 (Briggs, Schwab & Sramek) of *"Synthesis Imaging in Radio Astronomy II"*

Imaging the data

We now have the sampled and weighted visibilities, $\boldsymbol{V^W}$.

In order to efficiently make an image of our target sky brightness distribution, $\boldsymbol{I}$, we need to take the Fourier transform of this using Fast Fourier Transforms (FFTs).

This requires the $\boldsymbol{V^W}$ data to be gridded on to a regular grid.

This is done by convolution with some suitable gridding function*. Leaving us ultimately with some weighted and gridded visibilities which can be FFT'd to give us our dirty image $\boldsymbol{I^D}$.

*Discussion of gridding algorithms is a little beyond the scope of this workshop, please check the references at the start of this talk for more information.

CLEAN-ing

We've seen that our dirty image $\boldsymbol{I^D}$ is the sky brightness distribution convolved with the synthesised beam $\boldsymbol{B}$.

To get a better representation of the sky brightness distribution we need to remove the artefacts introduced by $\boldsymbol{B}$. To achieve this we use CLEAN*

*CLEAN is not an acronym, but it is usually captialised. I think it is merely tradition at this point.

Simple CLEAN overview

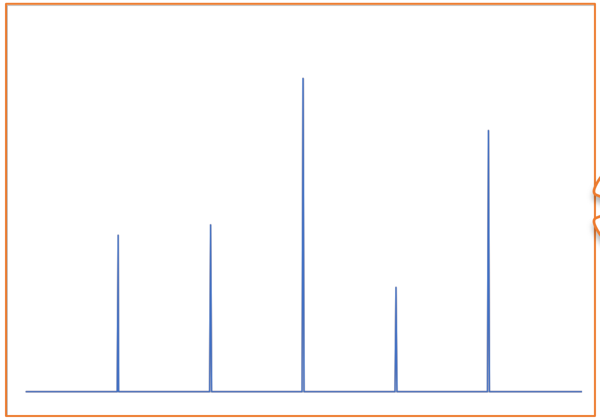
The Högbom (1974) CLEANing algorithm is the simplest CLEAN algorithm and is very illustrative of how CLEAN works in general.

In words the Högbom algorithm works as follows:

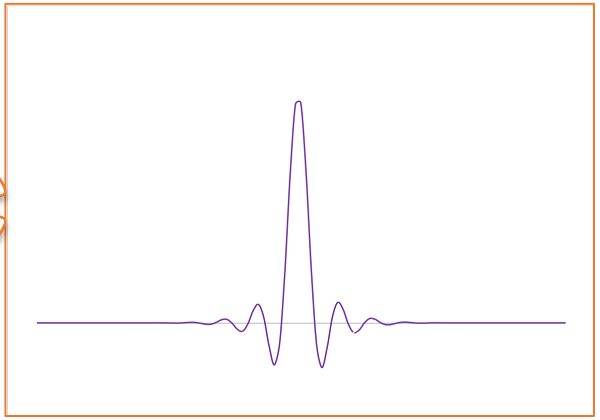
1) Find the magnitude and position of peak emission in the dirty image.
2) Subtract from the dirty image the dirty beam, B, scaled by some gain value (i.e. 0.1). Creating a 'residual' image.
3) Note the position and magnitude subtracted as a point in a model.
4) Repeat 1-3 until a user defined threshold is reached, either some noise limit (in the residual) or a given number of iterations.
5) Convolve the final model with an idealised beam. I.e. a beam based on the interferometer if it was a huge single dish.

Or in a 2D example on the next page

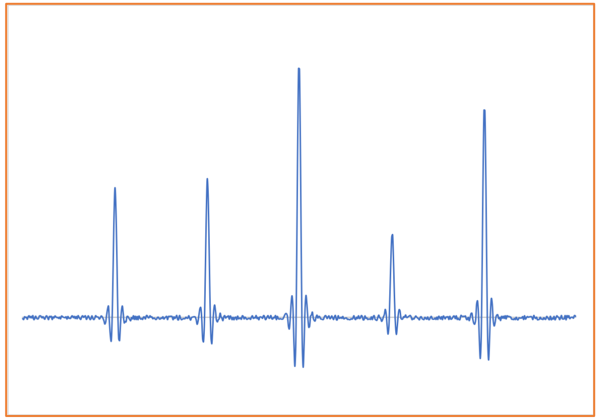
Högbom 2D example



True sky of 5 point sources

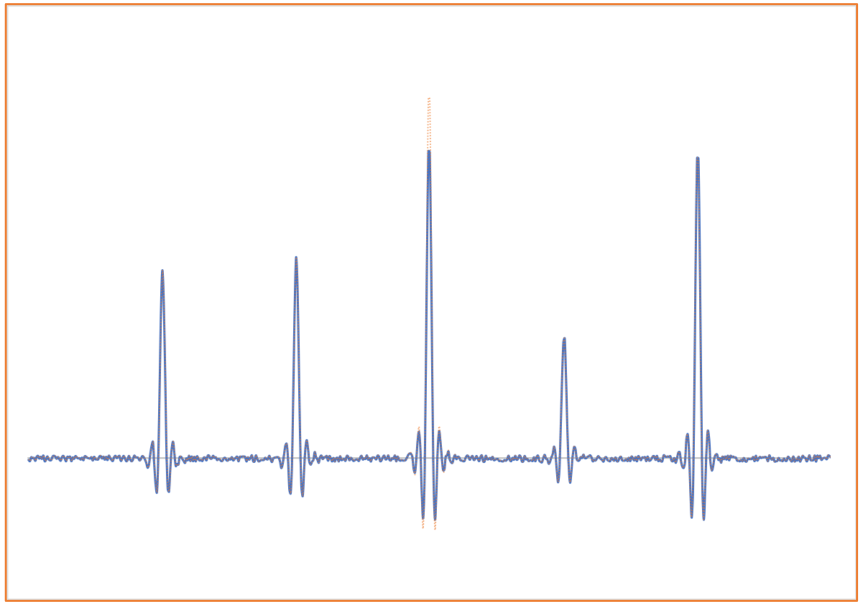


Dirty Beam (DB)

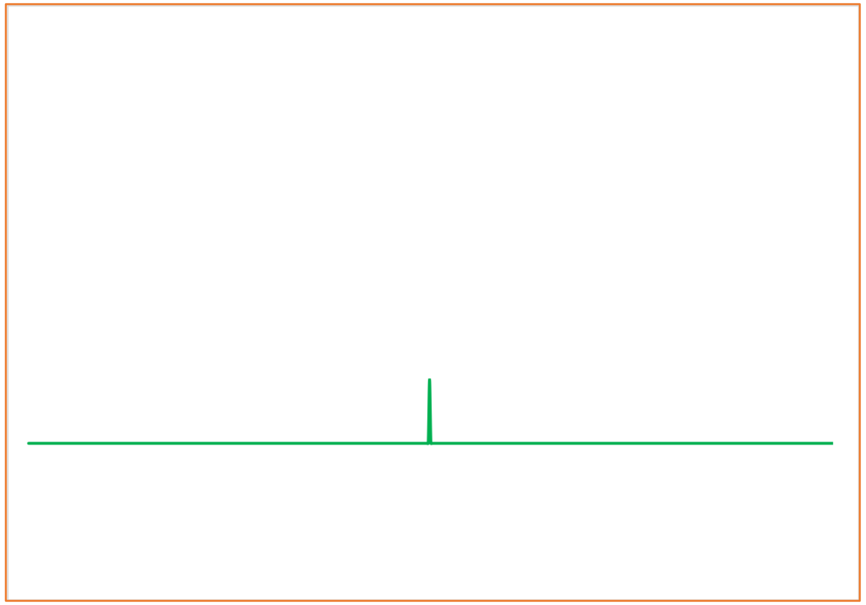


Dirty Image + noise, (DI)

Iteration 1

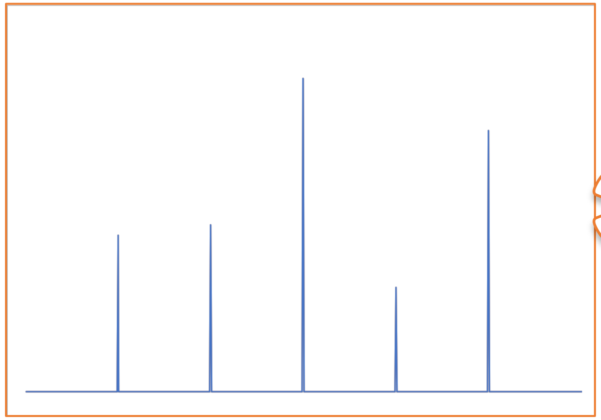


Residual Image after subtracting DB from peak in DI

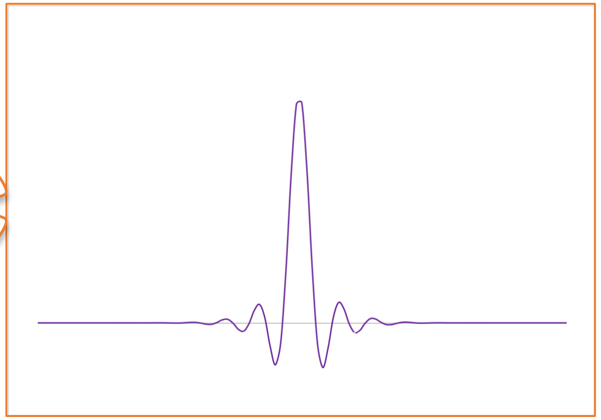


CLEAN components in Model Image

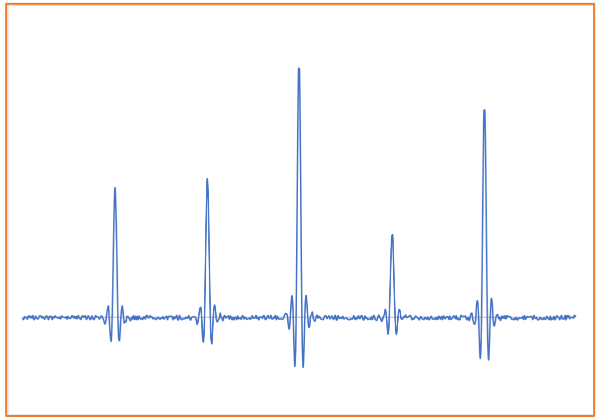
Högbom 2D example



True sky of 5 point sources

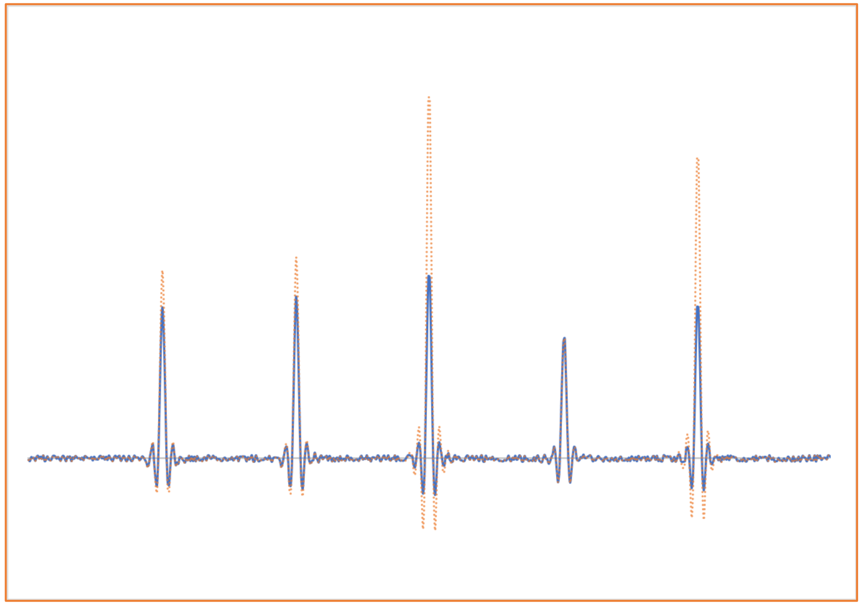


Dirty Beam (DB)

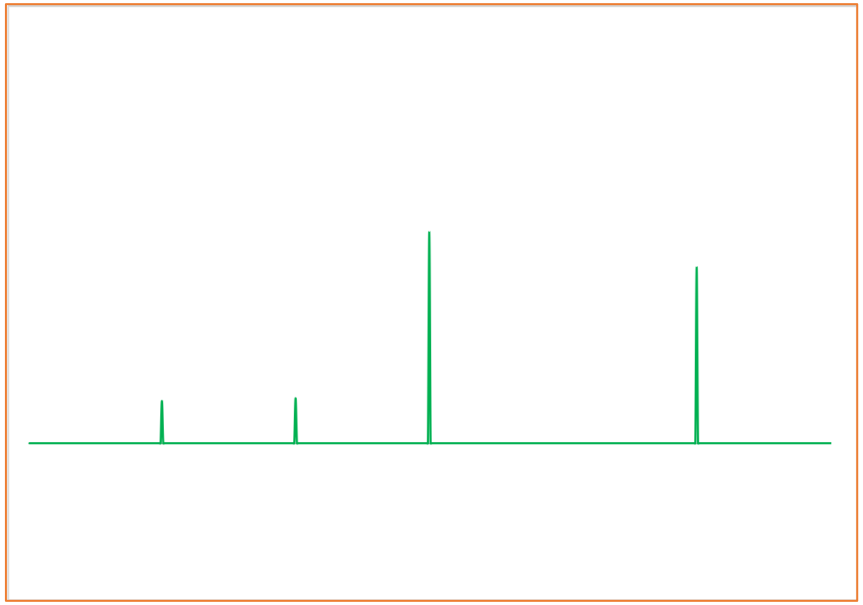


Dirty Image + noise, (DI)

Iteration *n*

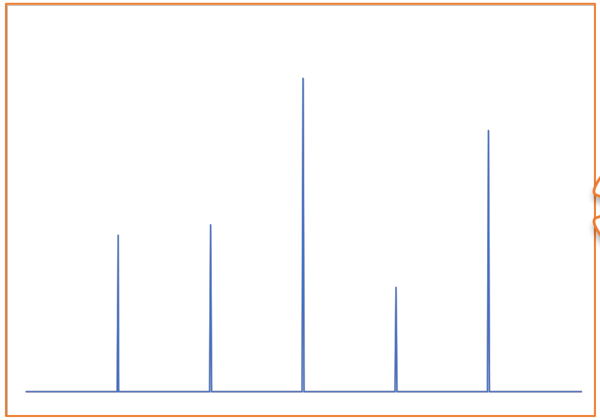


Residual Image after subtracting several DBs from located peaks in DI

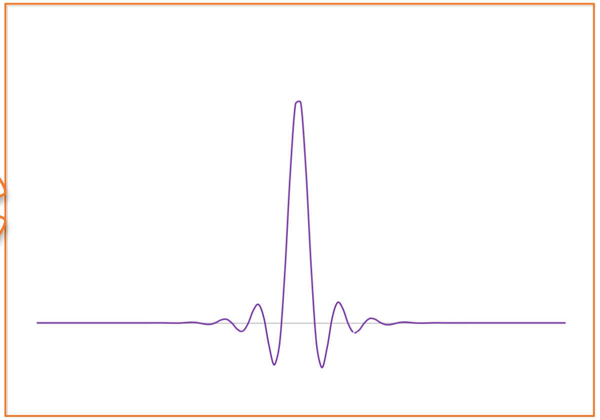


CLEAN components in Model Image

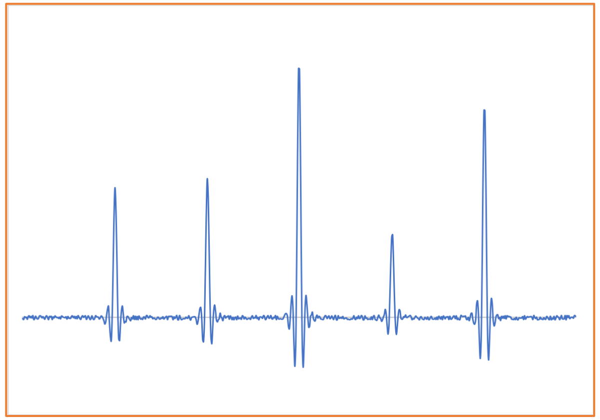
Högbom 2D example



True sky of 5 point sources

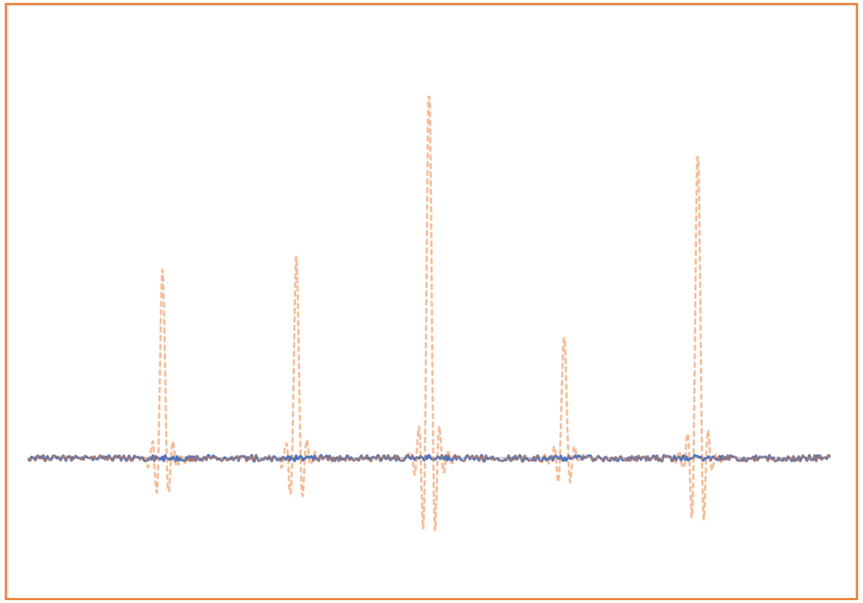


Dirty Beam (DB)

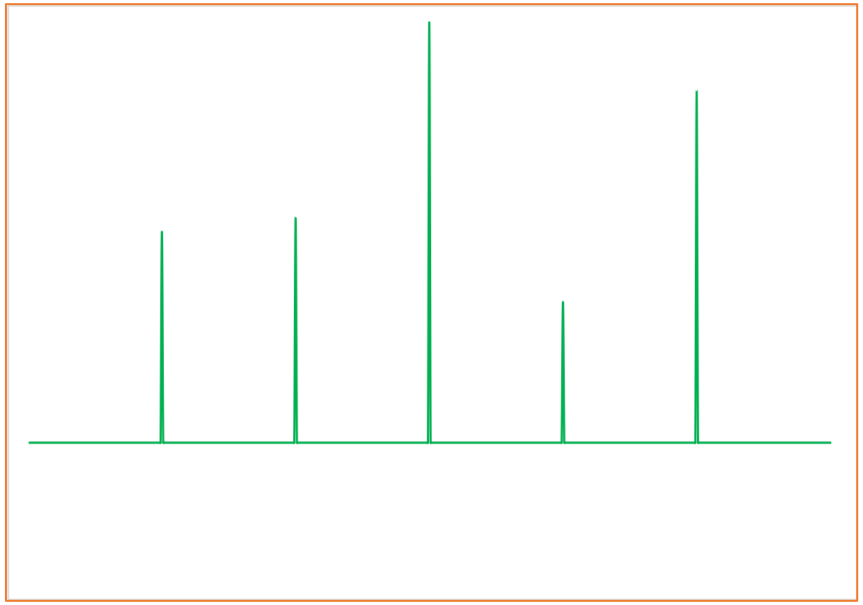


Dirty Image + noise, (DI)

Iteration *FINAL*

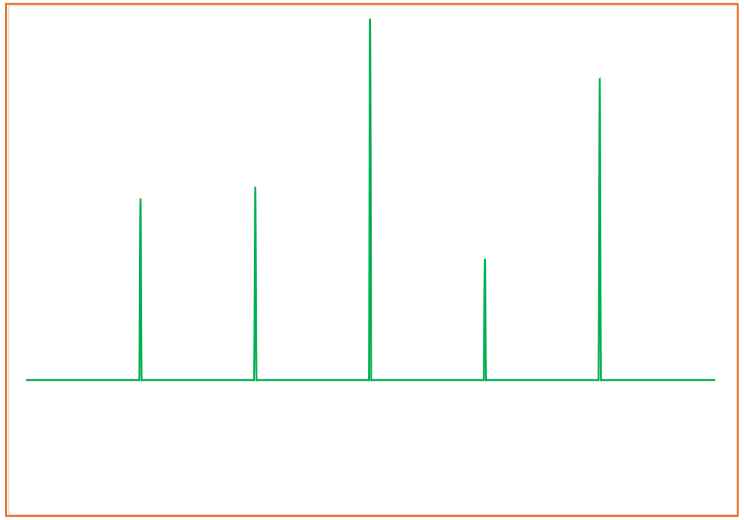


Residual Image after subtracting enough DBs from located peaks in DI until threshold met.



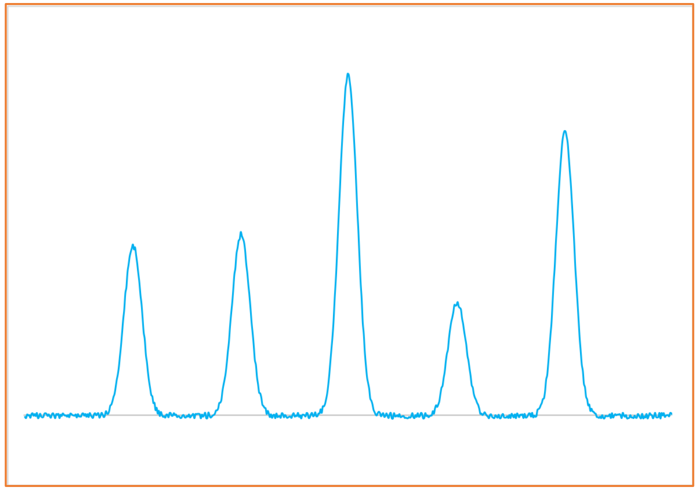
CLEAN components in model Image after final CLEAN loop

Högbom 2D example



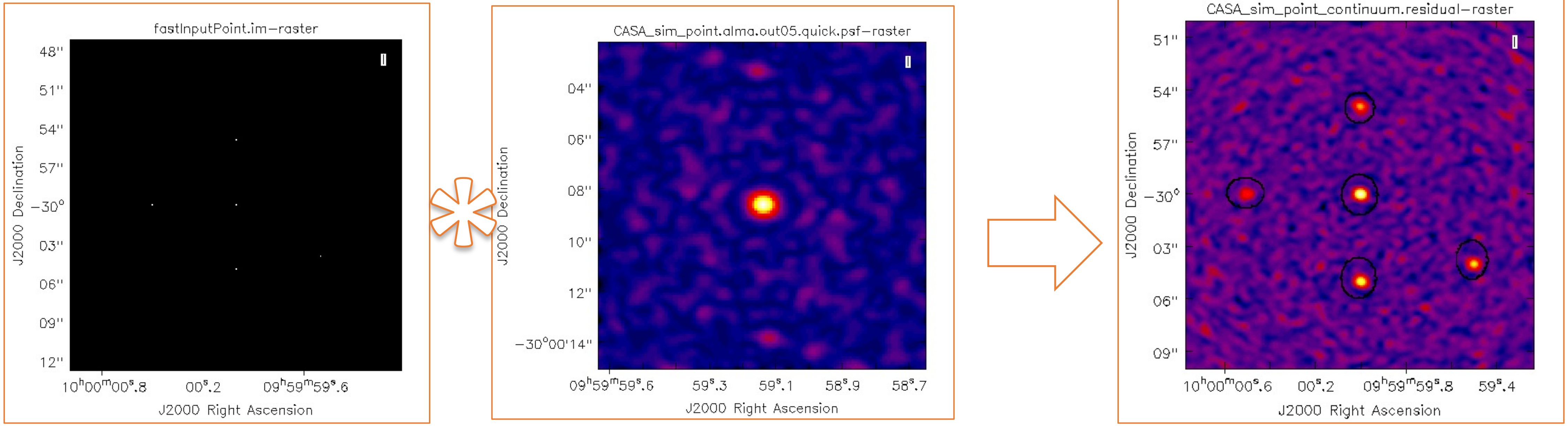
CLEAN components in model Image after final CLEAN loop

Idealized beam



Final reconstructed image

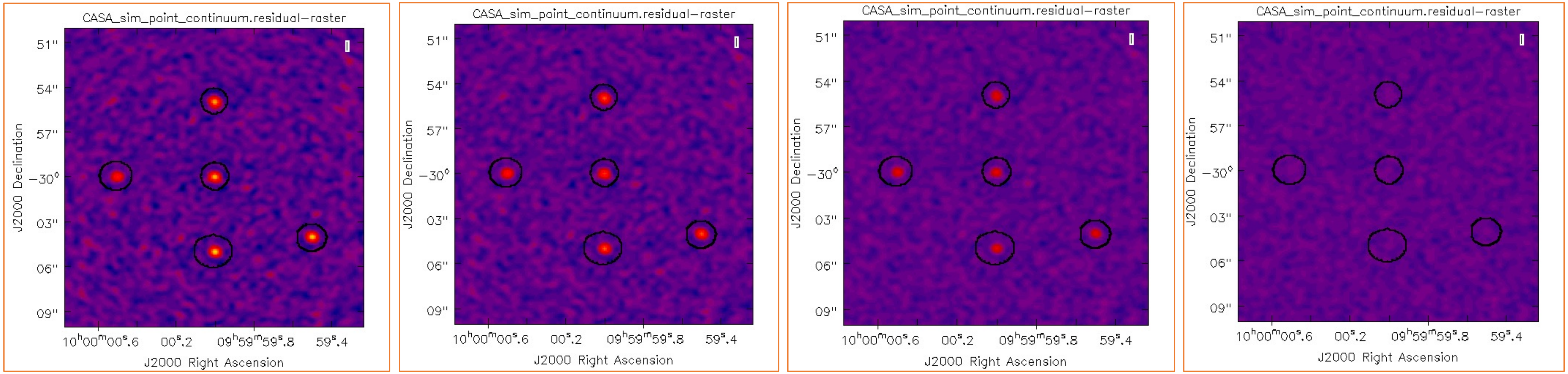
... and in 3D



True sky of 5 point sources

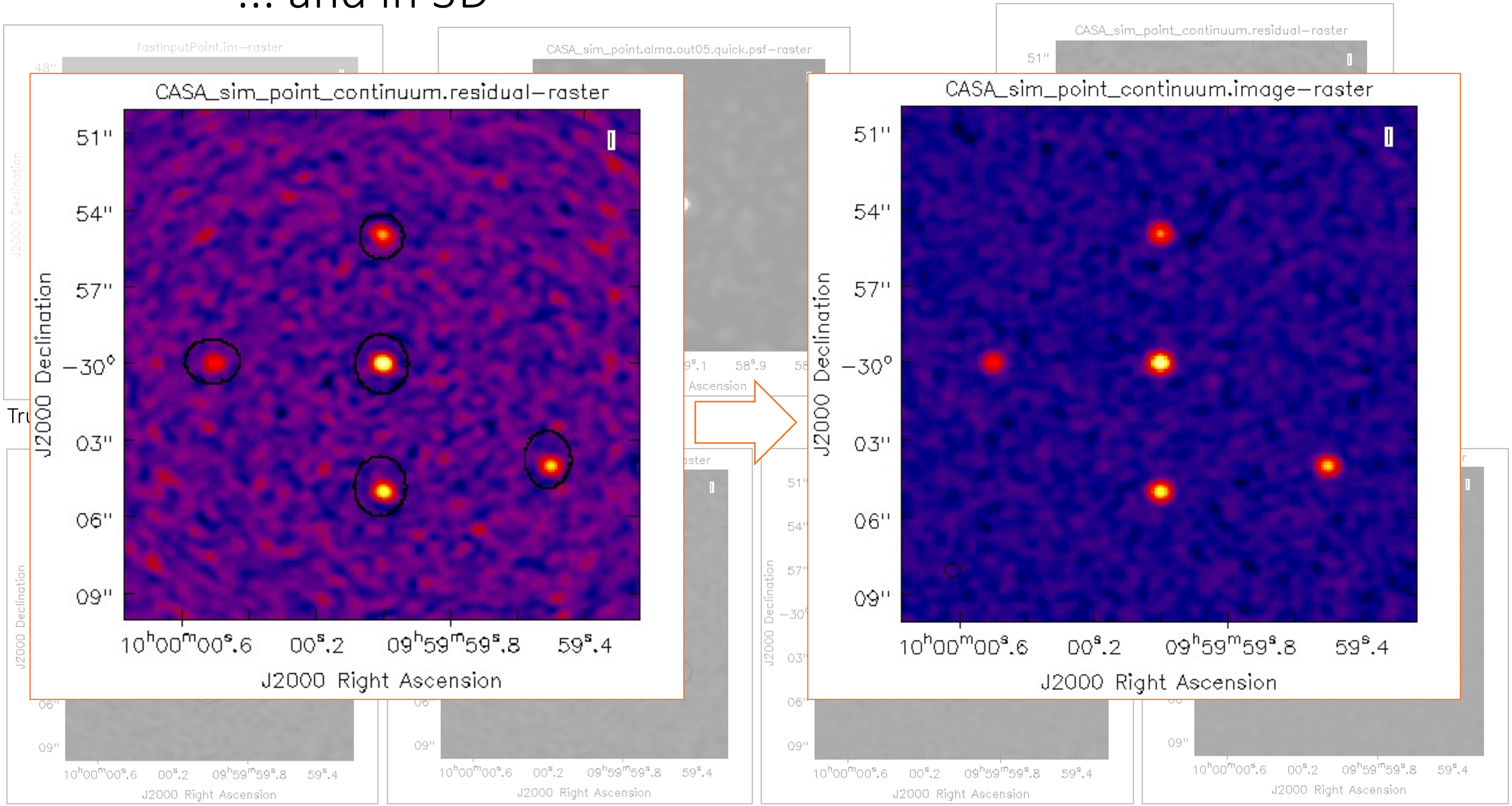
Dirty Beam (DB)

Dirty Image + noise, (DI)



→ Increasing CLEAN cycles →

... and in 3D


fastInputPoint.im–raster
CASA_sim_point.alma.out05.quick.psf–raster
CASA_sim_point_continuum.residual–raster
CASA_sim_point_continuum.residual–raster
CASA_sim_point_continuum.image–raster
51''
54''
57''
−30°
03''
06''
09''
J2000 Declination
10h00m00s.6
00s.2
09h59m59s.8
59s.4
J2000 Right Ascension

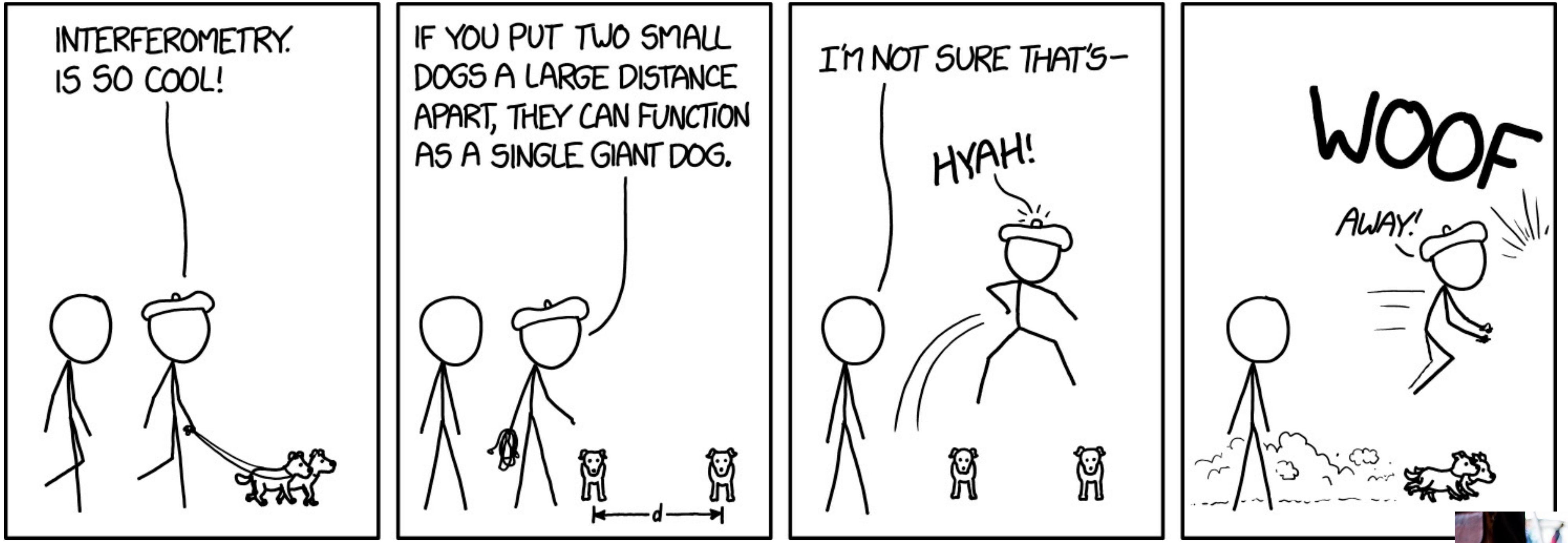
And after all that you’ll have a nice image/data cube from which you can actually do some Science!

Further Reading

The slides from this talk are based on the fundamentals of interferometry which are explained in detail across:

- *"Interferometry and Synthesis in Radio Astronomy"* - Thompson, Moran & Swenson
- *"Synthesis Imaging in Radio Astronomy II"* – NRAO
- *"An introduction to Radio Astronomy"* – Burke and Graham-Smith (4th edition out now as Burke, Graham-Smith & Wilkinson)
- *"Tools of Radio Astronomy"* – Wilson, Rohfls & Hüttemeister
- *"The CASA Cookbook"* – Ott & Kern et al.

¡Cheers!



Credit - xkcd.com

¿Questions?